

Numerical Partial Differential Equations II

Discontinuous Galerkin Methods

Homework 3

(Due: Jun 12, 2005)

Consider the 1-D second-order elliptic problem,

$$\frac{d^2}{dx^2}u = f(x), \quad x \in (0, 2),$$

with the boundary condition,

$$u(0) = u_0, \quad u(2) = u_2.$$

Write a code to solve the problem using Discontinuous Galerkin methods with Legendre polynomials as basis functions.

1. Solve the problem with various grid sizes and

$$f(x) = x, \quad u_0 = 0, \quad u_2 = 2.$$

- Compute the L^2 -errors of the numerical solutions. What is the rate of convergence?
- Choose spaces violating the compatibility condition. Is the problem solvable? Why?
- Discuss the results.

2. (Interface Problem) Now, consider the elliptic with a discontinuous f ,

$$f(x) = \begin{cases} 0 & 0 \leq x < 1 \\ 1 & 1 \leq x \leq 2 \end{cases},$$

boundary conditions,

$$u_0 = 0, \quad u_2 = 2,$$

and jump conditions

$$u(1^+) - u(1^-) = \frac{1}{2}, \quad u_x(1^+) - u_x(1^-) = \frac{1}{2}.$$

Solve the problem with various grid sizes.

- Compute the L^2 -errors of the numerical solutions. What is the rate of convergence?
- Change $f(x)$ to

$$f(x) = \begin{cases} 0 & 0 \leq x < 1.1 \\ 1 & 1.1 \leq x \leq 2 \end{cases},$$

(without jump conditions) what is the rate of convergence?

- Discuss the results.