Advanced Calculus (I)

WEN-CHING LIEN

Department of Mathematics
National Cheng Kung University

Definition

A number x is a least element of a set $E \subset \mathbf{R}$ if and only if $x \in E$ and $x \leq a$ for all $a \in E$

1.2 well-Ordering Principle

Definition

A number x is a least element of a set $E \subset \mathbf{R}$ if and only if $x \in E$ and $x \leq a$ for all $a \in E$

Postulate 3: [Well-Ordering Principle]

Every nonempty subset of $\mathbf{N}$ has a least element.

Postulate 3: [Well-Ordering Principle]

Every nonempty subset of $\mathbf{N}$ has a least element.

Theorem

Suppose for each $n \in \mathbf{N}$ that $A(n)$ is a proposition (i.e., a verbal statement or formula) which satisfies the two properties:

(i) $A(1)$ is true.

(ii) For every $n \in \mathbf{N}$ for which $A(n)$ is true, $A(n+1)$ is also true.

Then $A(n)$ is true for all $n \in \mathbf{N}$.

Theorem

Suppose for each $n \in \mathbf{N}$ that A(n) is a proposition (i.e., a verbal statement or formula) which satisfies the two properties:

(i)A(1) is true.

(ii) For every $n \in \mathbf{N}$ for which A(n) is true, A(n+1) is also true.

Then A(n) is true for all $n \in \mathbf{N}$.

Theorem

Suppose for each $n \in \mathbf{N}$ that A(n) is a proposition (i.e., a verbal statement or formula) which satisfies the two properties:

(i)A(1) is true.

(ii) For every $n \in \mathbf{N}$ for which A(n) is true, A(n+1) is also true.

Then A(n) is true for all $n \in \mathbf{N}$.

Theorem

Suppose for each $n \in \mathbf{N}$ that A(n) is a proposition (i.e., a verbal statement or formula) which satisfies the two properties:

(i)A(1) is true.

(ii) For every $n \in \mathbf{N}$ for which A(n) is true, A(n+1) is also true.

Then A(n) is true for all $n \in \mathbf{N}$.

Theorem (Binomial Formula)

If $a, b \in \mathbf{R}$ and $n \in \mathbf{N}$, then

$$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^k$$

Theorem (Binomial Formula)

If $a, b \in \mathbf{R}$ and $n \in \mathbf{N}$, then

$$(a+b)^n = \sum_{k=0}^{n} \begin{pmatrix} n \\ k \end{pmatrix} a^{n-k} b^k$$

Thank you.