Advanced Calculus (II)

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10.5: Connected Sets

Definition (10.53)

Let X be a metric space.
(i) A pair of nonempty open sets U, V in X is said to *separate* X if and only if $X = U \cup V$ and $U \cap V = \emptyset$.

(ii) X is said to be *connected* if and only if X cannot be separated by any pair of open sets U, V.

Definition (10.54)

Let X be a metric space and $E \subseteq X$.
(i) A set $U \subseteq E$ is said to be *relatively open* in E if and only if there is a set V open in X such that $U = E \cap V$.

(ii) A set $A \subseteq E$ is said to be *relatively closed* in E if and only if there is a set C closed in X such that $A = E \cap C$.

Remark (10.55)

Let $E \subseteq X$. If there exists a pair of open sets A, B in X which separate E; i.e., if $E \subseteq A \cup B$, $A \cap B = \emptyset$, $A \cap E \neq \emptyset$, and $B \cap E \neq \emptyset$, then E is not connected.

Theorem (10.56)

A subset E of $\mathbf{R}$ is connected if and only if E is an interval.

Theorem (10.57)

Let $E \subseteq X$. If there exist sets U, V, relatively open in E such that $U \cap V = \emptyset$, $E = U \cup V$, $U \neq \emptyset$, and $V \neq \emptyset$, then there is a pair of open sets A, B that separate E.

Thank you.