Advanced Calculus (I)

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2.4 Cauchy Sequences

Definition

A sequence of points $x_n \in \mathbf{R}$ is said to be *Cauchy* if and only if for every $\epsilon > 0$ there is an $N \in \mathbf{N}$ such that

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Thank you.