Advanced Calculus (I)

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4.2 Differentiability Theorems

Theorem

Let f and g be real function and $\alpha \in \mathbf{R}$. If f and g are differentiable at a, then $f+g$, αf, $f \cdot g$, and (when $g(a) \neq 0$) $\frac{f}{g}$ are all differentiable at a. In fact,

$$(f+g)'(a) = f'(a) + g'(a),$$

$$(\alpha f)'(a) = \alpha f'(a),$$

$$(f \cdot g)'(a) = g(a)f'(a) + f(a)g'(a).$$

$$\left(\frac{f}{g}\right)'(a) = \frac{g(a)f'(a) - f(a)g'(a)}{g^2(a)}$$

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Theorem (Chain Rule)

Let f and g be real functions. If f is differentiable at a and g is differentiable at f(a), then $g \circ f$ is differentiable at a with

$$(g \circ f)'(a) = g'(f(a))f'(a).$$

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Thank you.