Advanced Calculus (I)

WEN-CHING LIEN

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Definition

Let $S = \sum_{k=1}^{\infty} a_k$ be an infinite series whose terms a_k belong to **R** (i) The *partial sums of S of order n* are the numbers defined, for each $n \in \mathbf{N}$, by

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WEN-CHING LIEN Advanced Calculus (I)

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S is said to *converge* if and only if its sequence of partial sums $\{s_n\}$ converges to some $s \in \mathbf{R}$ as $n \to \infty$; i.e., for every $\epsilon > 0$ there is an $N \in \mathbf{N}$ such that $n \ge N$ implies that $|s_n - s| < \epsilon$. In this case we shall write

$$\sum_{k=1}^{\infty} a_k = s$$

and called s the *sum*, or *value*, of the series $\sum_{k=1}^{\infty} a_k$

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WEN-CHING LIEN Advanced Calculus (I)

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WEN-CHING LIEN Advanced Calculus (I)

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Example: [Harmonic Series]

Prove that the sequence $\frac{1}{k}$ converges but the series $\sum_{k=1}^{\infty} \frac{1}{k}$ diverges to $+\infty$.

WEN-CHING LIEN Advanced Calculus (I)

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The sequence 1/k converges to zero (by Example 2.2). On the other hand, by the Comparison Theorem for Integrals,

$$\sum_{k=1}^{n} \frac{1}{k} \ge \sum_{k=1}^{n} \int_{k}^{k+1} \frac{1}{x} dx = \int_{1}^{n+1} \frac{1}{x} dx = \log(n+1).$$

We conclude that $s_n \to \infty$ as $n \to \infty$. \Box

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Theorem (Divergence Test)

Let $\{a_k\}_{k \in \mathbb{N}}$ be a sequence of real numbers. If a_k does not converge to zero, then the series $\sum_{k=1}^{\infty} a_k$ diverges.

WEN-CHING LIEN Advanced Calculus (I)

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WEN-CHING LIEN Advanced Calculus (I)

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Suppose to the contrary that $\sum_{k=1}^{\infty} a_k$ converges to some $s \in \mathbf{R}$. By definition, the sequence of partial sums $s_n := \sum_{k=1}^n a_k$ converges to s as $n \to \infty$. Therefore, $a_k = s_k - s_{k-1} \to s - s = 0$ as $k \to \infty$, a contradiction.

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Theorem (Telescopic Test)

If $\{a_k\}$ is a convergent real sequence, then

$$\sum_{k=1}^{\infty}(a_k-a_{k+1})=a_1-\lim_{k\to\infty}a_k.$$

WEN-CHING LIEN Advanced Calculus (I)

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WEN-CHING LIEN Advanced Calculus (I)

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WEN-CHING LIEN Advanced Calculus (I)

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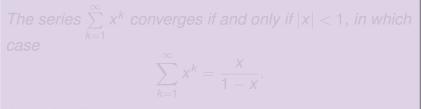
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Theorem (Geometric Series)



(see also Exercise 1.)

WEN-CHING LIEN Advanced Calculus (I)

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Theorem (Geometric Series)

The series $\sum_{k=1}^{\infty} x^k$ converges if and only if |x| < 1, in which case

$$\sum_{k=1}^{\infty} x^k = \frac{x}{1-x}.$$

(see also Exercise 1.)

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If $|x| \ge 1$, then $\sum_{k=1}^{\infty} x^k$ diverges by the Divergence Test. If |x| < 1, then set $s_n = \sum_{k=1}^n x^k$ and observe by the telescoping that

$$(1-x)S_n = (1-x)(x+x^2+\dots+x^n) = x+x^2+\dots+x^n-x^2-x^3-\dots-x^{n+1} = x-x^{n+1}.$$

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$$s_n = \frac{x}{1-x} - \frac{x^{n+1}}{1-x}$$

for all $n \in \mathbb{N}$. Since $x^{n+1} \to 0$ as $n \to \infty$ for all |x| < 1 (see Example 2.20), we conclude that $s_n \to \frac{x}{(1-x)}$ as $n \to \infty$.

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Theorem (Cauchy Criterion)

Let $\{a_k\}$ be a real sequence. Then the infinite series $\sum_{k=1}^{\infty} a_k$ converges if and only if for every $\epsilon > 0$ there is an $N \in \mathbb{N}$ such that

$$m > n \ge N$$
 imply $\left| \sum_{k=1}^{m} a_k \right| < \epsilon$

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$$s_m-s_{n-1}=\sum_{k=n}^m a_k$$

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Let $\{a_k\}$ and $\{b_k\}$ be real sequences. If $\sum_{k=1} a_k$ and $\sum_{k=1} b_k$ are convergent series, then

$$\sum_{k=1}^{\infty} (a_k + b_k) = \sum_{k=1}^{\infty} a_k + \sum_{k=1}^{\infty} b_k$$

and

 $\sum_{k=1}^{\infty} (\alpha a_k) = \alpha \sum_{k=1}^{\infty} a_k$

for any $\alpha \in \mathbf{R}$.

WEN-CHING LIEN Advanced Calculus (I)

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Exmple:

(1)
$$\sum_{k=1}^{\infty} \frac{1}{k(k+1)}$$

(2)
$$\sum_{k=1}^{\infty} \left(1 - \frac{1}{k}\right)^{k}$$

WEN-CHING LIEN Advanced Calculus (I)

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