1. Derive the Adams-Bashforth method of order 3
   \[ y_{n+1} = y_n + \frac{h}{12} \left[ 23y_n' - 16y_{n-1}' + 5y_{n-2}' \right] \]

2. Show that second order Runge-Kutta formulas of the form
   \[ y_{n+1} = y_n + h \left[ \gamma_1 f(x_n, y_n) + \gamma_2 f(x_n + \alpha h, y_n + \beta h f(x_n, y_n)) \right] \]
   should impose these conditions
   \[ \gamma_1 + \gamma_2 = 1, \quad \gamma_2 \alpha = \frac{1}{2}, \quad \gamma_2 \beta = \frac{1}{2} \]

3. Show that the Trapezoidal method
   \[ y_{n+1} = y_n + \frac{h}{2} \left[ f(x_n, y_n) + f(x_{n+1}, y_{n+1}) \right] \]
   is absolutely stable.

4. Implement the Gauss-Seidel iterative method as a MATLAB function.

5. Show that the general residual correction method
   \[ r^{(k)} = b - Ax^{(k)} \]
   \[ Ne^{(k)} = r^{(k)} \]
   \[ x^{(k+1)} = x^{(k)} + e^{(k)} \]
   is exactly the same as the method
   \[ Nx^{(k+1)} = b + Px^{(k)}. \]
   Here, \( A = N - P \).

6. Find the Cholesky factorization \( A = LL^T \) for the matrix
   \[ A = \begin{pmatrix} 1 & 2 & 4 \\ 2 & 13 & 14 \\ 4 & 14 & 21 \end{pmatrix} \]

7. Suppose that \( P_2(x) \in \Pi_2 \) interpolates \( f \) at the three nodes \( x_0 = x_1 - h, x_1, \) and \( x_2 = x_1 + h. \) Show that
   \[ P'(x_1) = \frac{f(x_1 + h) - f(x_1 - h)}{2h} \]
   and
   \[ f'(x_1) = P'_2(x_1) - \frac{h^2}{6} f'''(c_2) \]
   with \( x_1 - h \leq c_2 \leq x_1 + h. \) Note that \( \Pi_2 \) is the set of polynomials of degree \( \leq 2 \)

8. Work out the 3-node Gaussian numerical integration formula
   \[ \int_{-1}^{1} f(x) \, dx \approx \omega_1 f(x_1) + \omega_2 f(x_2) + \omega_3 f(x_3) \]
   by finding the nodes \( x_i \) and the weights \( \omega_i \) so that the formula is exact for polynomials of degree less than or equal to 5. Hint: Legendre polynomial \( P_3(x) = 5x^3 - 3x. \)