1. (10%) Construct the second Lagrange interpolating polynomial for \( f(x) = 2/x \), using the nodes \( x_0 = 1, \ x_1 = 1.5, \) and \( x_2 = 4 \).

2. (10%) A natural cubic spline \( S \) on \([0, 2]\) is defined by

\[
S(x) = \begin{cases} 
S_0(x) = 1 + 2x - x^2, & \text{if } 0 \leq x < 1, \\
S_1(x) = 2 + b(x - 1) + c(x - 1)^2 + d(x - 1)^3, & \text{if } 1 \leq x \leq 2.
\end{cases}
\]

Find \( b, c, \) and \( d \).

3. (10%) Approximate the following integral using Gaussian quadrature with \( n = 2 \):

\[
\int_1^{1.5} x \ln x \, dx.
\]

4. (12%) Find a factorization of the form \( A = LL^t \) for the matrix

\[
A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}
\]

5. (10%) Find the first two iterations of the Jacobi method, using \( x^{(0)} = 0 \), for the linear system

\[
\begin{bmatrix} 10 & -1 & 0 \\ -1 & 10 & -2 \\ 0 & -2 & 10 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 9 \\ 7 \\ 6 \end{bmatrix}.
\]

6. (12%) Show that the vector \( x^* \) is a solution to the positive definite linear system \( Ax = b \) if and only if \( x^* \) minimize

\[
g(x) = \frac{1}{2} \langle x, Ax \rangle - \langle x, b \rangle.
\]

7. (12%) Show that a symmetric matrix \( A \) is positive definite if and only if all the eigenvalues of \( A \) are positive.

8. (12%) What does the Newton’s method (for nonlinear system of equations) reduce to for the linear system \( Ax = b \).

9. (12%) Derive the formula for the Forward Difference method for the parabolic partial differential equation

\[
\frac{\partial u}{\partial t}(x,t) = \frac{\partial^2 u}{\partial x^2}(x,t), \quad 0 < x < 1, \quad t > 0,
\]

subject to the conditions

\[
u(0, t) = u(1, t) = 0, \quad t > 0,
\]

\[
u(x, 0) = f(x), \quad 0 \leq x \leq 1.
\]