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In a class of 10 students, five are to be chosen and seated in a row for a picture. How many such linear arrangements are possible?

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For an integer $n \geq 0$, $n$ factorial (denoted $n!$) is defined by

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$$n! = (n)(n−1)(n−2)\cdots(3)(2)(1),$$

for all $n \geq 1$. 
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1.2: Permutations

Definition (1.2)
Given a collection of \( n \) distinct objects, any (linear) arrangement of these objects is called a \textbf{permutation} of the collection.

Formula and Notation
If there are \( n \) distinct objects and \( r \) is an integer, with \( 1 \leq r \leq n \), then by the rule of product, the number of permutations of size \( r \) for the \( n \) objects is

\[
P(n, r) = n \times (n-1) \times (n-2) \times \cdots \times (n-r+1)
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**Example (1.10)**

The number of permutation of the letters in the word COMPUTER is $8!$. If only five of the letters are used, the number of permutations (of size 5) is $P(n, r) = 8!/(8 - 5)! = 8!/3! = 6720$. If repetition of letters are allowed, the number of possible 12-letter sequences is $8^{12} = 6.872 \times 10^{10}$. 

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Permutations with repeated objects:

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The MASSASAUGA is a brown and white venomous snake indigenous to North America. Arranging all of the letters in MASSASAUGA, we find that there are

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\frac{10!}{4!3!1!1!1!1!} = 25200 \text{ possible arrangements.}
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If all four A’s are together? For this question, we considered all arrangements of the seven symbols AAAA(one symbol), S, S, S, M, U, G. So, the answer is \[
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Example (1.16)

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