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For primitive statement $p$ and $q$, construct a truth table for each of the following compound statements.

a) $\neg p \lor q$

b) $p \rightarrow q$

Here we see that the corresponding truth tables for two statement $\neg p \lor q$ and $p \rightarrow q$ are exactly the same.

Definition (2.2)

Two statement $s_1$, $s_2$ are said to be **logically equivalent**, and we write $s_1 \iff s_2$, when the statement $s_1$ is true (respectively, false) if and only if the statement $s_2$ is true (respectively, false).

From the table, we know that $\neg p \lor q \iff p \rightarrow q$. 
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Example (2.8)

Construct a truth table for each of the following statements

a) \( \neg(p \land q) \),

b) \( \neg p \lor \neg q \),

c) \( \neg(p \lor q) \),

d) \( \neg p \land \neg q \),

where \( p, q \) are primitive statements.

Here, a crucial difference emerges: The negation of the conjunction of two primitive statement \( p, q \) results in the disjunction of their negations \( \neg p, \neg q \), whereas the negation of the disjunction of these same statements \( p, q \) is logically equivalent to the conjunction of their negations \( \neg p, \neg q \).
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For any primitive statements $p, q, r$, any tautology $T_0$, and any contradiction $F_0$,

1) $\neg
\neg p \iff p \quad \text{Laws of Double Negation}$

2) $\neg (p \lor q) \iff \neg p \land \neg q \quad \text{DeMorgan’s Laws}$
   $\neg (p \land q) \iff \neg p \lor \neg q$

3) $p \lor q \iff q \lor p \quad \text{Commutative Laws}$
   $p \land q \iff q \land p$

4) $p \lor (q \lor r) \iff (p \lor q) \lor r \quad \text{Associative Laws}$
   $p \land (q \land r) \iff (p \land q) \land r$

5) $p \lor (q \land r) \iff (p \lor q) \land (p \lor r) \quad \text{Distributive Laws}$
   $p \land (q \lor r) \iff (p \land q) \lor (p \land r)$
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6) \( p \lor p \iff p \)  \hspace{1cm} \text{Idempotent Laws}
   \( p \land p \iff p \)

7) \( p \lor F_0 \iff p \)  \hspace{1cm} \text{Identity Laws}
   \( p \land T_0 \iff p \)

8) \( p \lor \neg p \iff T_0 \)  \hspace{1cm} \text{Inverse Laws}
   \( p \land \neg p \iff F_0 \)

9) \( p \lor T_0 \iff T_0 \)  \hspace{1cm} \text{Domination Laws}
   \( p \land F_0 \iff F_0 \)

10) \( p \lor (p \land q) \iff p \)  \hspace{1cm} \text{Absorption Laws}
    \( p \land (p \lor q) \iff p \)

Remark:

- \( T_0 = \text{tautology} \)
- \( F_0 = \text{contradiction} \)
Definition (2.3 Dual)

Let $s$ be a statement. If $s$ contains no logical connectives other than $\land$ and $\lor$, then the **dual** of $s$, denoted $s^d$, is the statement obtained from $s$ by replacing each occurrence of $\land$ and $\lor$ by $\lor$ and $\land$, respectively, and each occurrence of $T_0$ and $F_0$ by $F_0$ and $T_0$, respectively.

Theorem (2.1 The Principle of Duality)

Let $s$ and $t$ be statements that contain no logical connectives other than $\land$ and $\lor$. If $s \iff t$, then $s^d \iff t^d$. 
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Theorem (2.1 The Principle of Duality)

Let $s$ and $t$ be statements that contain no logical connectives other than $\land$ and $\lor$. If $s \iff t$, then $s^d \iff t^d$. 
2.2: Logical Equivalence: The Laws of Logic

Two substitution rules:

1) Suppose that the compound statement $P$ is a tautology. If $p$ is a *primitive* statement that appears in $P$ and we replace each occurrence of $p$ by the *same* statement $q$, then the resulting compound statement $P_1$ is also a tautology.

2) Let $P$ be a compound statement where $p$ is an arbitrary statement that appears in $P$, and let $q$ be a statement such that $q \iff p$. Suppose that in $P$ we replace one or more occurrences of $p$ by $q$. Then this replacement yields the compound statement $P_1$. Under these circumstances $P_1 \iff P$. 
Example (2.12)

Negate and simplify the compound statement \((p \lor q) \rightarrow r\).

Example (2.15)

Verify the following compound statements are logically equivalent:

a) \((p \rightarrow q) \iff (\neg q \rightarrow \neg p)\)

b) \((q \rightarrow p) \iff (\neg p \rightarrow \neg q)\)

The statement \(\neg q \rightarrow \neg p\) is called the **contrapositive** of the implication \(p \rightarrow q\). The statement \(q \rightarrow p\) is called the **converse** of \(p \rightarrow q\); \(\neg p \rightarrow \neg q\) is called the **inverse** of \(p \rightarrow q\).
Simplification of compound statements:

Example (2.16, 2.17, 2.18)

Let $p$, $q$, $r$, $t$ denote primitive statements. Simplify each of the following compound statements:

a) $(p \lor q) \land \neg(\neg p \land q)$

b) $\neg[\neg[(p \lor q) \land r] \lor \neg q]$  

c) $(p \lor q \lor r) \land (p \lor t \lor \neg q) \land (p \lor \neg t \lor r)$