1. (a) Find an equation of the sphere that passes through the point \( (6, -2, 3) \) and has center \((-1, 2, 1)\).
(b) Find the curve in which this sphere intersects the yz-plane.
(c) Find the center and radius of the sphere
\[ x^2 + y^2 + z^2 - 8x + 2y + 6z + 1 = 0 \]
2. Copy the vectors in the figure and use them to draw each of the following vectors.
(a) \( \mathbf{a} + \mathbf{b} \) (b) \( \mathbf{a} - \mathbf{b} \) (c) \(-\frac{1}{2}\mathbf{a}\) (d) \(2\mathbf{a} + \mathbf{b}\)

\[ \mathbf{a} \]
\[ \mathbf{b} \]
3. If \( \mathbf{u} \) and \( \mathbf{v} \) are the vectors shown in the figure, find \( \mathbf{u} \cdot \mathbf{v} \) and \( |\mathbf{u} \times \mathbf{v}|\). Is \( \mathbf{u} \times \mathbf{v} \) directed into the page or out of it?

\[ |\mathbf{v}| = 3 \]
\[ 45^\circ \]
\[ |\mathbf{u}| = 2 \]
4. Calculate the given quantity if
\[ \mathbf{a} = \mathbf{i} + \mathbf{j} - 2\mathbf{k} \quad \mathbf{b} = 3\mathbf{i} - 2\mathbf{j} + \mathbf{k} \quad \mathbf{c} = \mathbf{j} - 5\mathbf{k} \]
(a) \(2\mathbf{a} + 3\mathbf{b}\) 
(b) \(|\mathbf{b}|\) 
(c) \(\mathbf{a} \cdot \mathbf{b}\) 
(d) \(\mathbf{a} \times \mathbf{b}\) 
(e) \(|\mathbf{b} \times \mathbf{c}|\) 
(f) \(\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})\) 
(g) \(\mathbf{c} \times \mathbf{c}\) 
(h) \(\mathbf{a} \times (\mathbf{b} \times \mathbf{c})\) 
(i) \(\text{comp}_\mathbf{a}\mathbf{b}\) 
(j) \(\text{proj}_\mathbf{a}\mathbf{b}\) 
(k) The angle between \(\mathbf{a}\) and \(\mathbf{b}\) (correct to the nearest degree)
5. Find the values of \(x\) such that the vectors \(\langle 3, 2, x\rangle\) and \(\langle 2x, 4, x\rangle\) are orthogonal.
6. Find two unit vectors that are orthogonal to both \(\mathbf{j} + 2\mathbf{k}\) and \(\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}\).
7. Suppose that \(\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w}) = 2\). Find
(a) \((\mathbf{u} \times \mathbf{v}) \cdot \mathbf{w}\) 
(b) \(\mathbf{u} \cdot (\mathbf{w} \times \mathbf{v})\) 
(c) \(\mathbf{v} \cdot (\mathbf{u} \times \mathbf{w})\) 
(d) \((\mathbf{u} \times \mathbf{v}) \cdot \mathbf{v}\) 
8. Show that if \(\mathbf{a}\), \(\mathbf{b}\), and \(\mathbf{c}\) are in \(V_3\), then
\[ (\mathbf{a} \times \mathbf{b}) \cdot [(\mathbf{b} \times \mathbf{c}) \times (\mathbf{c} \times \mathbf{a})] = [\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})]^2 \]
9. Find the acute angle between two diagonals of a cube.
10. Given the points \(A(1, 0, 1), B(2, 3, 0), C(-1, 1, 4),\) and \(D(0, 3, 2)\), find the volume of the parallelepiped with adjacent edges \(AB, AC,\) and \(AD\).
11. (a) Find a vector perpendicular to the plane through the points \(A(1, 0, 0), B(2, 0, -1),\) and \(C(1, 4, 3)\).
(b) Find the area of triangle \(ABC\).
12. A constant force \(\mathbf{F} = 3\mathbf{i} + 5\mathbf{j} + 10\mathbf{k}\) moves an object along the line segment from \((1, 0, 2)\) to \((5, 3, 8)\). Find the work done if the distance is measured in meters and the force in newtons.
13. A boat is pulled onto shore using two ropes, as shown in the diagram. If a force of 255 N is needed, find the magnitude of the force in each rope.

14. Find the formula for the distance from the origin to the line with parametric equations
\[ x = 0 - t, \quad y = 1 + 3t, \quad z = 4t \]
15–17 Find parametric equations for the line.
15. The line through \((4, -1, 2)\) and \((1, 1, 5)\)
16. The line through \((1, 0, -1)\) and parallel to the line \(\frac{1}{2}(x - 4) = \frac{1}{2}y = z + 2\)
17. The line through \((-2, 2, 4)\) and perpendicular to the plane \(2x - y + 5z = 12\)
18–20 Find an equation of the plane.
18. The plane through \((2, 1, 0)\) and parallel to \(x + 4y - 3z = 1\)
19. The plane through \((3, -1, 1), (4, 0, 2),\) and \((6, 3, 1)\)
20. The plane through \((1, 2, -2)\) that contains the line \(x = 2t, \quad y = 3 - t, \quad z = 1 + 3t\)
21. Find the point in which the line with parametric equations
\[ x = 2 - t, \quad y = 1 + 3t, \quad z = 4t \]
intersects the plane \(2x - y + z = 2\).
22. Find the distance from the origin to the line \(x = 1 + t, \quad y = 2 - t, \quad z = -1 + 2t\).
23. Determine whether the lines given by the symmetric equations
\[ \frac{x - 1}{2} = \frac{y - 2}{3} = \frac{z - 3}{4} \]
and
\[ \frac{x + 1}{6} = \frac{y - 3}{4} = \frac{z + 5}{2} \]
are parallel, skew, or intersecting.

24. (a) Show that the planes \( x + y - z = 1 \) and \( 2x - 3y + 4z = 5 \) are neither parallel nor perpendicular.
(b) Find, correct to the nearest degree, the angle between these planes.

25. Find the distance between the planes \( 3x + y - 4z = 2 \) and \( 3x + y - 4z = 24 \).

26–34 Identify and sketch the graph of each surface.
26. \( x = 3 \)  27. \( x = z \)
28. \( y = z^2 \)  29. \( x^2 = y^2 + 4z^2 \)
30. \( 4x - y + 2z = 4 \)
31. \( -4x^2 + y^2 - 4z^2 = 4 \)
32. \( y^2 + z^2 = 1 + x^2 \)
33. \( 4x^2 + 4y^2 - 8y + z^2 = 0 \)
34. \( x = y^2 + z^2 - 2y - 4z + 5 \)

35. An ellipsoid is created by rotating the ellipse \( 4x^2 + y^2 = 16 \) about the \( x \)-axis. Find an equation of the ellipsoid.

36. A surface consists of all points \( P \) such that the distance from \( P \) to the plane \( y = 1 \) is twice the distance from \( P \) to the point \((0, -1, 0)\). Find an equation for this surface and identify it.

37. (a) Sketch the curve with vector function \( r(t) = ti + \cos \pi t \, j + \sin \pi t \, k \) \( t \geq 0 \)
(b) Find \( r'(t) \) and \( r''(t) \).

38. Let \( r(t) = \left( \sqrt{2-t}, e^{-1} / t, \ln(t+1) \right) \).
(a) Find the domain of \( r \).
(b) Find \( \lim_{t \to 0} r(t) \).
(c) Find \( r'(t) \).

39. Find a vector function that represents the curve of intersection of the cylinder \( x^2 + y^2 = 16 \) and the plane \( x + z = 5 \).

40. Find parametric equations for the tangent line to the curve \( x = 2 \sin t, y = 2 \sin 2t, z = 2 \sin 3t \) at the point \((1, \sqrt{3}, 2)\). Graph the curve and the tangent line on a common screen.

41. If \( r(t) = t^2 \, i + t \cos \pi t \, j + \sin \pi t \, k \), evaluate \( \int_0^1 r(t) \, dt \).

42. Let \( C \) be the curve with equations \( x = 2 - t^3, y = 2t - 1, z = \ln t \). Find (a) the point where \( C \) intersects the \( xz \)-plane, (b) parametric equations of the tangent line at \((1, 1, 0)\), and (c) an equation of the normal plane to \( C \) at \((1, 1, 0)\).

43. Use Simpson’s Rule with \( n = 6 \) to estimate the length of the arc of the curve with equations \( x = t^2, y = t^3, z = t^4, 0 \leq t \leq 3 \).

44. Find the length of the curve \( r(t) = (2t^{3/2}, \cos 2t, \sin 2t) \), \( 0 \leq t \leq 1 \).

45. The helix \( r(t) = \cos t \, i + \sin t \, j + t \, k \) intersects the curve \( r_2(t) = (1 + t) \, i + t^2 \, j + t^3 \, k \) at the point \((1, 0, 0)\). Find the angle of intersection of these curves.

46. Reparametrize the curve \( r(t) = e^t \, i + e^t \sin t \, j + e^t \cos t \, k \) with respect to arc length measured from the point \((1, 0, 1)\) in the direction of increasing \( t \).

47. For the curve given by \( r(t) = \left( \frac{1}{2} t^3, \frac{1}{2} t^2, t \right) \), find (a) the unit tangent vector, (b) the unit normal vector, and (c) the curvature.

48. Find the curvature of the ellipse \( x = 3 \cos t, y = 4 \sin t \) at the points \((3, 0)\) and \((0, 4)\).

49. Find the curvature of the curve \( y = x^4 \) at the point \((1, 1)\).

50. Find an equation of the osculating circle of the curve \( y = x^4 - x^2 \) at the origin. Graph both the curve and its osculating circle.

51. A particle moves with position function \( r(t) = t \, \ln t \, i + t \, j + e^{-t} \, k \). Find the velocity, speed, and acceleration of the particle.

52. A particle starts at the origin with initial velocity \( i - j + 3 \, k \). Its acceleration is \( a(t) = 6t \, i + 12t^2 \, j - 6t \, k \). Find its position function.

53. An athlete throws a shot at an angle of 45° to the horizontal at an initial speed of 43 ft/s. It leaves his hand 7 ft above the ground.
(a) Where is the shot 2 seconds later?
(b) How high does the shot go?
(c) Where does the shot land?

54. Find the tangential and normal components of the acceleration vector of a particle with position function \( r(t) = t \, i + 2t \, j + t^2 \, k \).

55. Find the curvature of the curve with parametric equations
\[
\begin{align*}
x &= \int_0^t \sin \left( \frac{\pi}{2} \theta^2 \right) \, d\theta \\
y &= \int_0^t \cos \left( \frac{\pi}{2} \theta^2 \right) \, d\theta
\end{align*}
\]