Numerical Analysis 11/19/2010

1. Derive the asymptotic error formula (10 %)

$$\tilde{E}_n^T(f) \approx \frac{-h^2}{12}\left[f'(b) - f'(a)\right]$$

for the composite trapezoidal rule.

2. Find the linear least squares approximation to $f(x) = \sin(x)$ on $[0, \frac{1}{2}\pi]$. (10 %)

3. Find the linear near–minimax approximation for $f(x) = e^x$ on $[-1, 1]$. (10 %)
Hint: Chebyshev polynomial $T_2(x) = 2x^2 - 1$.

4. For an integer $n \geq 0$, define the Chebyshev polynomials (10 %)

$$T_n(x) = \cos(n \cos^{-1} x), \qquad -1 \leq x \leq 1$$

Show that if $m \neq n$, then

$$\int_{-1}^{1} T_m(x)\, T_n(x) \frac{1}{\sqrt{1-x^2}}\, dx = 0$$

5. Find the natural cubic spline that interpolates the data points $\{(0,1), (1,1), (2,5)\}$. (10 %)
Hint: $S_0''(x) = z_1(x-1)$, $S_2''(x) = z_1(2-x)$, where $z_1 = S''(1)$.

6. Show that if $x_0, \ldots, x_n$ are n+1 distinct nodes and f is sufficiently smooth, then (10 %)

$$f[x_0, x_1, \ldots, x_n] = \frac{f^{(n)}(\xi)}{n!}$$

for some ξ.

7. Find the polynomial in Newton's form that interpolates the data

x	-2	-1	0	1	2	3
y	-5	1	1	1	7	25

8. Show that, for the secant method, the iterates x_n satisfies (10 %)

$$\alpha - x_{n+1} = (\alpha - x_n)(\alpha - x_{n-1}) \left[\frac{-f''(\xi_n)}{2f'(\zeta_m)}\right]$$

where α is a root of $f(x)$.