Numerical Analysis

1. (20%) Determine c_1, c_2, x_1, and x_2 so that the integration formula

$$\int_{-1}^{1} f(x)\,dx \approx c_1 f(x_1) + c_2 f(x_2)$$

gives the exact result whenever $f(x)$ is a polynomial of degree 3 or less.

2. (20%) Describe the Gauss–Seidel method for solving a linear system $A\mathbf{x} = \mathbf{b}$, and find the first two iterations of this method for the system with

$$A = \begin{bmatrix} 3 & -1 & 1 \\ 3 & 6 & 2 \\ 3 & 3 & 7 \end{bmatrix} \qquad \mathbf{b} = \begin{bmatrix} 1 \\ 0 \\ 4 \end{bmatrix}$$

using $\mathbf{x}^{(0)} = 0$.

3. (20%) Factor the matrix

$$A = \begin{bmatrix} 2 & -1 & 1 \\ 3 & 3 & 9 \\ 3 & 3 & 5 \end{bmatrix}$$

into the LU decomposition with $l_{ii} = 1$ for all i.

4. (20%) Write down a computer program in C or FORTRAN to apply Taylor's method of order 2 to solve the initial–value problem

$$y' = y - t^2 + 1, \quad 0 \le t \le 2, \quad y(0) = 0.5$$

5. (20%) Write an algorithm to apply the Newton's method for finding the solution of the nonlinear system

$$5x_1^2 - x_2^2 = 0$$
$$x_2 - 0.25(\sin x_1 + \cos x_2) = 0$$

using $(\frac{1}{4}, \frac{1}{4})^t$ as the starting point.