

# The 7th Trilateral Meeting (Australia - Italy - Taiwan) on Nonlinear PDEs and Applications and The 27th Annual Meeting on Differential Equations and Related Topics,

Date: Jan. 23 (Wed) 2019 ~ Jan. 28 (Mon) 2019

Venue: Ger-Jyh Hall, Small lecture hall, College of Science

National Cheng Kung University (Cheng Kung Campus)

國立成功大學成功校區，理化大樓，格致廳小講堂

Website: [http://www.ncts.ntu.edu.tw/events\\_2\\_detail.php?nid=210](http://www.ncts.ntu.edu.tw/events_2_detail.php?nid=210)

<http://www.math.ncku.edu.tw/news/news.php>

<http://www.math.ncku.edu.tw/~fang/>

Online Registration: <https://goo.gl/forms/ToLtgwHU5KR1xwlv2>

## Title and Abstract:

01/23 (Wednesday)

Neil Trudinger video of ICM 1970

"Piermarco Cannarsa" <cannarsa@mat.uniroma2.it>, uniroma2

Title: Mean field games with state constraints

Abstract: This talk will address deterministic mean field games for which agents are restricted in a closed domain of Euclidean space. In this case, the existence, uniqueness, and regularity of Nash equilibria cannot be deduced as for unrestricted state space because, for a large set of initial conditions, the uniqueness of solutions to the minimization problem which is solved by each agent is no longer guaranteed.

We will therefore attack the problem by considering a relaxed version of it, for which the existence of equilibria can be proved by set-valued fixed point arguments. We will then give a uniqueness result for such equilibria under a classical monotonicity assumption. Finally, we will analyze the regularity of the relaxed solution and show that it satisfies the typical first order PDE system of mean field games.

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"Xuan Duong" <xuan.duong@mq.edu.au>, Macquarie University

Title: Dispersive estimates for self-adjoint operators

Abstract: Let  $X$  be a metric space with a doubling measure satisfying  $\mu(B) \gtrsim r_B^n$  for any ball  $B$  with radius  $r_B$ . Let  $L$  be a non negative self-adjoint operator on  $L^2(X)$ . We assume that the semigroup  $e^{-tL}$  satisfies a Gaussian upper bound and that the flow  $e^{itL}$  satisfies a typical  $L^1 - L^\infty$  dispersive estimate of the form

$$\|e^{itL}\|_{L^1 \rightarrow L^\infty} \lesssim |t|^{-\frac{n}{2}}$$

Then we prove a similar  $L^1 - L^\infty$  dispersive estimate for a general class of flows  $e^{it\phi(L)}$ , with  $\phi(r)$  of power type near 0 and near  $\infty$ . In the case of fractional powers  $\phi(L) = L^\nu$ ,  $\nu \in (0,1)$ , we deduce dispersive estimates for  $e^{itL^\nu}$  with data in Sobolev, Besov, or Hardy spaces  $H_L^p$  with  $p \in (0,1]$ , associated to the operator  $L$ . This is a joint work with The Anh Bui, Piero D'Ancona and Detlef Müller.

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"Aldo Pratelli" <pratelli@math.fau.de>, University of Erlangen-Nurnberg

Title: On the isoperimetric problem with double density in  $R^N$ .

Abstract: We will discuss about the isoperimetric problem in  $R^N$  with double density. This means, one aims to minimize the perimeter of sets of given volume, but volume and perimeter are given by the integral of two different functions, called densities, over the set and its boundary respectively. As usual, the main questions are existence and regularity of minimizers. This problem has been deeply studied in the last decades, because of some interesting applications. The problem has primarily been studied with a single density, but the case of two different ones is particularly important, specially when the density of the perimeter also depends on the direction of the boundary. In this talk, we will give an overview of the main classical results and open questions, and we will concentrate on some recent developments. Parts of the talk are based on several joint papers with De Philippis, Franzina, Jachan, Morgan, Saracco.

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"(陳俊全) Chiun-Chuan Chen" <chchchen@math.ntu.edu.tw>, National Taiwan University

Title: A total mass estimate for the diffusive Lotka–Volterra system of competing species

Abstract: Using an elementary approach, we establish a new maximum principle for the diffusive Lotka–Volterra system of competing species in 1-dim case, which involves pointwise estimates of an elliptic equation consisting of the second derivative of one function, the first derivative of another function, and a quadratic nonlinear term. This maximum principle gives an a priori estimate for the total mass of the species in a traveling wave solution. Applying this estimate to the system of three competing species leads to a nonexistence theorem of traveling wave solutions.

"Jiakun Liu" <jiakunl@uow.edu.au>, University of Wollongong

Title: A boundary value problem for Monge-Ampere equations.

Abstract: In this talk, we will present a recent result on the global  $C^{2,\alpha}$  and  $W^{2,p}$  regularity for the Monge-Ampere equation subject to a natural boundary condition arising in optimal transportation.

This is a joint work with Shibing Chen and Xu-Jia Wang.

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"Ramiro A. Lafuente" <r.lafuente@uq.edu.au>, University of Queensland,

Title: Homogeneous Einstein manifolds via a cohomogeneity-one approach

Abstract: We establish non-existence results on non-compact homogeneous Einstein manifolds. The key idea in the proof is to consider non-transitive group actions on these spaces (more precisely, actions with cohomogeneity one), and to find geometric monotone quantities for the ODE that results from writing the Einstein equation in such a setting. As an application, we show that homogeneous Einstein metrics on Euclidean spaces are Einstein solvmanifolds. This is joint work with C. Bhm.

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01/24 (Thursday)

"Yihong Du" <ydu@une.edu.au>, University of New England

Title: The Dynamics of a Fisher-Kpp Nonlocal Diffusion Model with Free Boundaries

Abstract: We introduce and discuss a class of free boundary models with "nonlocal diffusion", which are natural extensions of the free boundary models considered by Du and Lin [SIAM J. Math. Anal., 2010] and elsewhere, where "local diffusion" is used to describe the population dispersal, with the free boundary representing the spreading front of the species. We show that this nonlocal problem has a unique solution defined for all time, and then examine its long-time dynamical behavior when the growth function is of Fisher-KPP type. We demonstrate that a spreading-vanishing dichotomy holds, though for the spreading-vanishing criteria significant differences arise from the well-known local diffusion model.

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"Stefano Bianchini" <bianchin@sissa.it>, SISSA, Trieste:

Title: A decomposition of vector fields in  $\mathbb{R}^{d+1}$

Abstract:

Given a vector field  $\rho(1, \mathbf{b}) \in L^1_{loc}(\mathbb{R}^+ \times \mathbb{R}^d, \mathbb{R}^{d+1})$  such that  $div_{t,x}(\rho(1, \mathbf{b}))$  is a measure, we consider the problem of uniqueness of the representation  $\eta$  of  $\rho(1, \mathbf{b}) \mathcal{L}^{d+1}$  as a superposition of characteristics  $\gamma: (t^-_\gamma, t^+_\gamma) \rightarrow \mathbb{R}^d$ ,  $\gamma'(t) = \mathbf{b}(t, \gamma(t))$ . We give conditions in terms of a local structure of the representation  $\eta$  on suitable sets in order to prove that there is a partition of  $\mathbb{R}^{d+1}$  into disjoint trajectories  $P_\alpha$ ,  $\alpha \in \mathcal{A}$ , such that the PDE

$$div_{t,x}(u \rho(1, \mathbf{b})) \in M(\mathbb{R}^{d+1}), \quad u \in L^\infty(\mathbb{R}^+ \times \mathbb{R}^d),$$

can be disintegrated into a family of ODEs along  $P_\alpha$  with measure r.h.s.. The decomposition  $P_\alpha$  is

essentially unique. We finally show that  $\mathbf{b} \in L^1_t(BV_x)_{loc}$ , particular, the renormalization property for nearly incompressible  $BV$  vector fields.

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"(吳恭儉)Kung-Chien Wu" <kcwu@mail.ncku.edu.tw>, National Cheng Kung University

Title: Spatial behavior of the solution to the Boltzmann equation with hard potentials

Abstract: The main goal of this talk is to understand the quantitative spatial decay of the solution to the Boltzmann equation with hard potentials for both linear and nonlinear problems.

For the nonlinear study, we get the spatial behavior by using the nonlinear weighted energy estimate. For the linear study, we get the quantitative space-time behavior under some slow velocity decay assumption, but without regularity assumption on the initial data. Both results reveal that hard sphere and hard potential models differ in their spatial behaviors. This is a joint work with Yu-Chu Lin and Haitao Wang.

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"Massimiliano Morini" <morini73@gmail.com>, <massimiliano.morini@unipr.it>, University of Parma

Title: Existence and uniqueness for anisotropic and crystalline mean curvature flows

Abstract: An existence and uniqueness result, up to fattening, for crystalline mean curvature flows with forcing and arbitrary (convex) mobilities, is proven. This is achieved by introducing a new notion of solution to the corresponding level set formulation. Such solutions satisfy a comparison principle and stability properties with respect to the approximation by suitably regularized problems. The results are valid in any dimension and for arbitrary, possibly unbounded, initial closed sets. As a result of our analysis, we deduce the convergence of a minimizing movement scheme proposed by Almgren, Taylor and Wang (1993), to a unique (up to fattening) "flat flow".

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"(王振男)Jenn-Nan Wang" <jnwang@math.ntu.edu.tw>, National Taiwan University

Title: Quantitative unique continuation for the fractional Schrödinger operator

Abstract: In this talk, I would like to discuss some quantitative uniqueness estimates related to the strong unique continuation property and the unique continuation at infinity for the fractional Schrödinger operator. These kinds of estimates are useful in understanding the local properties of the solution. For the classical Schrödinger operator, these estimates have been extensively studied and successfully applied to other problems. Recently, the study of the local properties of solutions to the fractional equation became possible thanks to the Caffarelli-Silvestre extension theorem. For the fractional Schrödinger operator, we are especially interested in the dependence of the estimates on the size of the potential. Besides of mathematical interests, fractional equations arise naturally from super-diffusion and can be used in modeling a lot of physical phenomena involving long jumps. -----

"Julie Clutterbuck" <julie.clutterbuck@monash.edu>, Monash University

Title: "The shape of the ground state for the Robin eigenvalue problem".

Abstract:

We consider the first eigenfunction of the Laplace operator with Robin boundary values. In the case of Neumann boundary values, the first eigenfunction is constant. In the case of Dirichlet boundary values, the first eigenfunction is log-concave. The Robin case is often considered to interpolate between these two, and so it is reasonable to ask whether the first Robin eigenfunction is also log-concave. We show that in general it is not, and classify the limited situations in which it is. This is joint work with Ben Andrews and Daniel Hauer.

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01/25 (Friday)

"Nicola Fusco" <n.fusco@unina.it>, University of Napoli:

Title: Asymptotic stability of the gradient flow of nonlocal energies

Abstract: I will discuss short time existence and long-time stability of a class of equations modeling the evolution of the interface between an elastic material and a material void, controlled by mass diffusion within the surface. These equations appear as the  $H^{-1}$ -gradient flow of an energy given by the sum of the area of the interface plus a nonlocal volume term. Our stability results are new even in the simplest case of the surface diffusion equation.

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"Zihua Guo" <zihua.guo@monash.edu>, Monash University

Title: Scattering for the 3D Gross-Pitaevskii equation

Abstract: We study the Cauchy problem for the 3D Gross-Pitaevskii equation. Global well-posedness in the natural energy space was proved by Gerard.

We prove scattering for small data in the same space with some additional angular regularity, and in particular in the radial case we obtain small energy scattering. The crucial ingredients are new generalized Strichartz estimates and some new observed "NULL" structures of the Gross-Pitaevskii equation after some normal form type transform. This is a joint work with Zaher Hani and Kenji Nakanishi.

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"Volker Elling" <velling@math.sinica.edu.tw>, Sinica

Title: Vortex cusps

Abstract: Vortex cusps are pairs of vortex sheets with opposite circulation that merge in a cusp. Such solutions are observed in engineering flows, for example vortex sheets in Mach reflection at a solid wall. We present modelling and numerics of vortex cusps, calculate the cusp exponent and discuss possible rigorous existence proofs.

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"Daniel Daners" <daniel.daners@sydney.edu.au>, University of Sydney,

Title: Degenerate periodic-parabolic evolution equations of logistic type

Abstract: We consider periodic-parabolic evolution equations with a logistic nonlinearity allowing spacial and temporal degeneration as a parameter becomes large. We characterize the existence and stability of positive periodic-parabolic solutions with the help of a parabolic maximum principle on non-cylindrical domains. This is joint work with Julian Lopez-Gomez.

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"(江金城) Jin-Cheng Jiang" <jcjiang@math.nthu.edu.tw>, National Tsing Hua University

Title: On the Cauchy problem for the Boltzmann equation

Abstract: In this talk, we will present some recent progress on the Cauchy problem for the Boltzmann equation. We will begin with the introduction of the Boltzmann equation, its connection with fluid dynamics. Then the result of the local well-posedness for the Cauchy problem of the non-cutoff Boltzmann equation in the weighted Sobolev space will be presented. The quasi-linear method instead of linearization method is used to prove the existence and the non-negativity of the solution.

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"(黄信元) Hsin-Yuan Huang" <hyhuang@math.nctu.edu.tw>, National Chiao Tung University

Title: On the Bubbling Solutions to the Liouville System

Abstract: In this talk, I will briefly introduce the recent developments on the Liouville system. The system is related to several models of Chemistry, Ecology and Physics. My recent result on the bubbling solutions will be present.

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## The 27th Annual Meeting on Differential Equations and Related Topics

01/26 (Saturday)

Plenary Talks:

"Kenji Nakanishi" <kenjinakanishi@gmail.com>, Kyoto University

Title: Randomized final data problem for the nonlinear Schrödinger and the Gross-Pitaevskii equations

Abstract: This is based on joint work with Takuto Yamamoto. We study large time behavior of solutions to the nonlinear Schrödinger equations with power-type interactions. For powers between the mass critical and the Fujita exponents, there exists a global solution asymptotic (at time infinity) to any free solution of finite mass in three or higher space dimensions. A scaling argument suggests that the uniqueness is a super-critical problem beyond the reach of standard perturbation arguments. Randomizing the final state, however, Murphy proved that one can almost surely find a unique asymptotic solution in a certain function space, if the power is above the Strauss exponent.

In this talk, it is shown that we can go slightly below the Strauss exponent by using another function space. In particular, it allows us to treat quadratic interactions in three space dimensions, which often appears in physical models. The same argument applies to the asymptotic form consisting of a plane wave and a linearized dispersive wave with finite energy for the defocusing cubic equation.

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Differential Equations:

"P-洪盟凱 John M. Hong" <jhong@math.ncu.edu.tw>, National Central University

Title: The Global Escape Phenomenon of Transonic Gas-like Fluids with Self-gravitation in Spherically Symmetric Space-times.

Abstract: In this talk, the global escape phenomenon of gas-like transonic fluids with self-gravitation in Spherically symmetric space-times is studied. The escape phenomenon is governed by an initial-boundary problem of one-dimensional compressible Euler-Poisson equations which form a mixed-type nonlinear partial differential system of balance laws. The compressible Euler-Poisson system is reformulated as a  $3 \times 3$  hyperbolic system of balance laws by the equations of fluid's density and the gravitational potential. The global existence to the shock wave solutions of fluid's density-momentum and the Lipschitz continuous solution to the gradient of potential, is established by a new version of generalized Glimm scheme (GGS for short). The new approximate solutions of generalized Riemann and boundary-Riemann problems, which are the building block of GGS, are constructed by the de-coupling process of fluid's quantities and potential's gradient. For the global boundedness of approximate solution by GGS, the key conditions to the momentum and potential's gradient on the boundary are provided. Finally, the modified wave interaction estimates are shown for the decay of Glimm functionals, which leads to the global existence of solutions.

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"PD-黃志強 Chih-Chiang Huang" <loveworldsteven@hotmail.com>, NCTS

Title: Traveling waves for the FitzHugh-Naumo system with monostable or bistable nonlinearity

Abstract: In this talk, we will study the FitzHugh-Naumo system (FHN) with monostable and bistable

nonlinearity, respectively. We also consider steady states of (FHN) in a bounded domain and traveling waves of (FHN) in a cylinder. By a variational method, we would like to construct traveling waves for a scalar equation and generalize this approach to an equation with a nonlocal term arising from the FitzHugh-Nagumo system (FHN). In addition, Turing patterns for (FHN) are discussed in the talk. This is a joint work with Chiun-Chuan Chen and Chao-Nien Chen.

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"AP-郭鴻文 Hung-Wen Kuo" <hwkuo@mail.ncku.edu.tw>, National Cheng Kung University

Title: Singularity of Free Molecular Flow in Bounded Domains

Abstract: We study the singularity of free molecular flow in the spherical symmetric domains. First, we show the singularities caused by the effects of the specular reflection boundary condition and the diffuse reflection boundary condition. Then we try to study whether the solution is smooth upon imposing some suitable conditions on initial data.

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Applied Math:

"AP-薛名成 Ming-Cheng Shiue" <mshiue@math.nctu.edu.tw>, National Chiao Tung University

Title: Data assimilation algorithms based on Synchronization of truth and models

Abstract: In this talk, we first recall continuous and discrete data assimilation algorithms that were proposed for designing finite-dimensional feedback controls for 2D Navier-Stokes equations. Then, two new nudging methods, hybrid nonlinear and delay-coordinate nudging are considered and studied. In the first part, hybrid nonlinear continuous data assimilation algorithms for Lorenz systems will be studied and presented. It is shown that the approximate solutions converge to the unknown reference solutions over time provided that the first or second variable of Lorenz systems is synchronized. This is a joint work with Yi Juna Du.

In the second part, two new continuous and discrete data assimilation algorithms for two-dimensional Navier-Stokes equations are presented and studied. The explicit use of present and past observations at each time step provides a way that new methods might outperform the old one, which was successfully tested for Lorenz 96 model.

In this talk, we will give preliminary results that provide sufficient conditions on the finite-dimensional spatial resolution of the collected data and observational measurements to make sure that the approximate solutions obtained from the new algorithms converge to the unknown reference solutions over time.

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"aP-樂美亨 Mei-Heng Yueh" <yue@ntnu.edu.tw>, National Taiwan Normal University

Title: Computational Conformal Geometry with Applications

Abstract: Computational conformal geometry is an interdisciplinary field based on the theories of



conformal geometry as well as computational algorithms. It has been widely applied to carry out 3D image processing tasks, such as surface resampling, remeshing, registration, rendering, and alignment. Especially when the geometry is complicated, a suitable parameterization of the surface can be used to simplify the shape of the domain. In this talk, I will introduce my recent works on the computation of surface parameterizations, and demonstrate some applications on computer graphics and visualization of medical images.

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"aP-許佳璵 Chia-Yu Hsu" <cyuhsu@fcu.edu.tw>, <chiahsutw@gmail.com>, Feng Chia University

Title: The Strategy for Schooling Pattern of Lampreys

Abstract: The numerical computational solutions for schooling of lampreys' swimming under some specific conditions, such as spacing in between fishes and initial body activation waves pattern next to or in front each one, are presented in this talk. The schooling pattern [1] in marine ecology is a common migration pattern for fishes of different swimming styles, such as carangiform of makrells, subcarangiform of salmonids or anguiliform of eels [2]. In particular, to school is one strategy to reduce energy consumption during migration [3], not to mention, to survival from predators [4]. In this talk, a model of multiple anguiliform swimmers, such as lamprey, is created to simulate the schooling pattern. The adaptive mesh refinement immersed boundary method is used to solve the numerical solution for the simulations. Moreover, is there possibility of synchronized schooling for paralleled multi-swimmers or what is the strategy to have the schooling pattern stabilized? Those are questions will be discussed in this talk.

Keywords: lamprey, schooling pattern, adaptive mesh refinement immersed boundary method  
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Plenary Talks:

"P-(尤釋賢) Shih-Hsien Yu", <matysh@nus.edu.sg>, National University of Singapore

Title: Heat equation with Bounded Variation heat conductivity

Abstract: In this talk, a new constructive procedure to establish the Green's function for heat equation with a BV function heat conductivity; and the pointwise structure of the Green's function will be established

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Differential Equations:

"AP-陳子軒 Chi-Hin Chan" <cchan@math.nctu.edu.tw>, National Chiao Tung University

Title: Anti-Thesis to the Stokes paradox on the hyperbolic plane.

Abstract: In this talk, we will discuss a recent result which is due to Chi Hin Chan and Magdalena Czubak in which we proved the existence of a nontrivial Stationary Navier-Stokes flow on an exterior domain of a hyperbolic plane which satisfies both the no-slip boundary condition and the finite Dirichlet norm property. This shows that there is no Stokes paradox in the hyperbolic plane setting.

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"aP-張覺心 Chueh-Hsin Chang" <changjuexin@thu.edu.tw>, Tung Hai University

Title: Attractive interaction of 2-species traveling waves for the 3 components competition-diffusion systems

Abstract: In this talk we consider the weak interaction between two traveling wave solutions of the three-species competition-diffusion systems. Each of the two traveling wave solutions has one trivial component (called trivial waves). By the invariant manifold theory and asymptotic behavior of kernels of linearized operators, we can prove the existence and instability of non-monotonic traveling wave solutions for three-species.

This is a joint work with Prof. Chiun-Chuan Chen and Prof. Shin-Ichiro Ei.

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"aP-陳逸昆 I-Kun Chen" <ikun.chen@gmail.com>, National Taiwan University

Title: Propagation of boundary-induced discontinuity in stationary radiative transfer and its application to the optical tomography

Abstract: We consider a boundary value problem of the stationary transport equation with the incoming boundary condition in two or three dimensional bounded convex domains. We discuss discontinuity of the solution to the boundary value problem arising from discontinuous incoming boundary data, which we call the boundary-induced discontinuity. In particular, we give two kinds of sufficient conditions on the incoming boundary data for the boundary-induced discontinuity. We propose a method to reconstruct attenuation coefficient from jumps in boundary measurements.

"aP-梁育豪 Yu-Hao Liang" <yhliang@nuk.edu.tw>, National University of Kaohsiung

Title: The effects of awareness on the epidemic models

Abstract: The rapid advance of technology has brought the communication between individuals more and more accessible and diverse. This also makes people have more chance to be aware of an infectious disease outbreak and hence reduce the risk of infection. In this talk, we will propose an epidemic model by taking into account the influence of awareness. In our model, a multiplex network for which the spreading of the disease and information occurs, respectively, in two different layers of networks, i.e., the physical network and the virtual network. In addition, these two diffusive processes are assumed to interact and affect each other. Some theoretical results on this model will be introduced. This is a joint work with Prof. Jonq Juang.

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Applied Math :

"aP-Maxim Solovchuk" <solovchuk@gmail.com>, National Health Research Institutes (NHRI)

Title: A Nonlinear Conservative System for Describing Highly Nonlinear Acoustic Waves in Heterogeneous Media

Abstract: A new system of hyperbolic PDEs capable of describing the nonlinear nature of acoustic fluctuations that propagate over inhomogeneous and heterogeneous fluid media is formulated. This novel system model is initially derived by using the traditional principles of nonlinear acoustics [1], i.e. the finite-amplitude methodology, to yield a general system for describing acoustic fluctuations from the Navier-Stokes-Fourier equations. Here, by incorporating the special substitution technique of [2], it is found that the classical result can be closed into a conservative system of nonlinear PDEs.

However, the resulting system is then found to be in a general form of the conservation laws, namely the capacitive-conservative differential form [3]. A closer look at the Rankine-Hugoniot relations that result from the system's associated flux function indicates that the system model is consistent with the physical expectations inside the acoustic regime. As a result, we extend the high-order shock-capturing numerical approach used in [4,5] so that the nonlinear nature of the acoustic propagation in heterogeneous fluid media (including shocks) can be captured without numerical artifacts while keeping any numerical dissipation to a minimum. To verify and illustrate the capabilities of the proposed nonlinear system model, one- and two-dimensional benchmark problems of the literature are studied [3,6]. Applications of the proposed system for the simulation of high intensity focused ultrasound treatment of liver cancer will be presented [7].

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"PD-黃韋強 Wei-Qiang Huang" <wqhuang@math.nctu.edu.tw>, National Chiao Tung University

Title: An Integrated Eigensolver for Graph Laplacian Eigenvalue Problem

Abstract: The eigenvalue problem of a graph Laplacian matrix arising from a simple, connected and undirected graph has been given more attention due to its extensive applications in the field of machine learning. The associated graph Laplacian matrix is symmetric, positive semi-definite, and is usually large and sparse. Computing some smallest positive eigenvalues and corresponding eigenvectors is often of interest for either clustering or dimensionality reduction.

However, its singularity makes the classical eigensolvers inefficient since we need to solve related linear systems. Moreover, for large-scaled networks from the real world, such as social media, transactional databases, and sensor systems, there are in general not only local connections. Therefore, it is usually time-consuming, or even unable, to directly find the matrix factorization for solving involved linear systems exactly. In this talk, we propose an inner-outer iterative eigensolver, iSIRA, based on the residual Arnoldi method together with an implicit remedy of the singularity and an effective deflation for convergent eigenvalues. Numerical experiments demonstrate that the integrated eigensolver outperforms the classical methods, especially in the case when the matrix factorization is not available.

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"PD-張正陽 Zhengyang Zhang" <zhengyang.zhang@math.nthu.edu.tw>, National Tsing Hua University

Title: A class of state-dependent delay differential equations and applications to forest growth

Abstract: We consider a state-dependent delay differential equation that describes the dynamics of a population of trees in a forest. This model comes from a size-structured population dynamical model. This class of state-dependent delay differential equation is compared with a computer model called SORTIE (which is an individual-based model). The main ingredient taken into account in both models is the competition for light between trees. The comparison suggests that state-dependent delay differential equations can help to understand the dynamics of forest, since we get pretty good fit to the SORTIE model. Therefore it makes sense to analyze the state-dependent delay differential equation. The second and third parts are devoted to the properties of the semi-flow generated by such a state-dependent delay differential equation and the boundedness and dissipativity of the solutions. In the last part, motivated by the nematode destruction in a pine forest, we construct a predator-prey system including the above

state-dependent delay differential equation and we present numerical simulations of this system in several cases and scenarios.

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"Dr-陳博源 Po-Yuan Chen" <pyrobertchen@gmail.com>, Medical Device Innovation Center, NCKU  
(成大前瞻醫療器材科技中心)

Title: Quadratic Adaptive Algorithm for Solving Cardiac Action Potential Models

Abstract: In this talk, I will give a short introduction to the numerical simulation of cardiac cell models and present a new adaptive integration method for computing cardiac action potential models. Time steps are adaptively chosen by solving a quadratic formula involving the first and second derivatives of the membrane action potential. To improve the numerical accuracy, we devise an extremum-locator (el) function to predict the local extremum when approaching the peak amplitude of the action potential. In addition, the time step restriction (tsr) technique is designed to limit the increase in time steps, and thus prevent the membrane potential from changing abruptly. The performance of the proposed method is tested using the Luo-Rudy phase 1 (LR1), dynamic (LR2), and human O'Hara-Rudy dynamic (ORd) ventricular action potential models, and the Courtemanche atrial model incorporating a Markov sodium channel model. Numerical experiments demonstrate that the action potential generated using the proposed method is more accurate than that using the traditional Hybrid method, especially near the peak region. The traditional Hybrid method may choose large time steps near to the peak region, and sometimes causes the action potential to become distorted. In contrast, the proposed new method chooses very fine time steps in the peak region, but large time steps in the smooth region, and the profiles are smoother and closer to the reference solution.

In the test on the stiff Markov ionic channel model, the Hybrid blows up if the allowable time step is set to be greater than 0.1 ms. In contrast, our method can adjust the time step size automatically, and is stable. Overall, the proposed method is more accurate than and as efficient as the traditional Hybrid method, especially for the human ORd model. The proposed method shows improvement for action potentials with a non-smooth morphology, and it needs further investigation to determine whether the method is helpful during propagation of the action potential. -----

01/27 (Sunday)

Plenary Talks:

"Giuseppe Mingione" <rosariomingione@gmail.com>, University of Parma:

Title: Lipschitz estimates for every taste

Abstract: I will focus on gradient estimates for solutions to non-homogeneous, possibly degenerate equations and systems. I will give a survey of results on Lipschitz estimates starting from the uniformly elliptic case, where linear and nonlinear potentials come into the play. I will then switch to the case of non-uniformly elliptic equations, where a new and optimal theory can be developed.

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Differential Equations:

"aP-林英杰 Ying-Chieh Lin" <linyj@nuk.edu.tw>, National University of Kaohsiung

Title : Concentration of source terms in generalized Glimm scheme for initial-boundary problem of nonlinear hyperbolic balance laws

Abstract: In this talk, we investigate the initial-boundary value problem for a nonlinear hyperbolic system of balance laws with sources  $a_x g$  and  $a_t h$ . To get the approximate solutions of our problem, we consider a version of generalized Riemann problem that concentrates the variation of  $a$  on a thin  $T$ -shaped region of each grid. A new version of Glimm scheme is introduced to construct the approximate solutions and its stability is proved by considering two types of conditions on  $a$ . Finally, we verify the consistency of the scheme and the entropy inequality to establish the global existence of entropy solutions.

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"PD-蘇承芳 Cheng-Fang Su" <scf1204@nctu.edu.tw>, National Chiao Tung University, Taiwan

Title: Incompressible inviscid limit of the viscous two-fluid model on expanding domains with general initial data

Abstract: This talk is about that the incompressible inviscid limit of the viscous two-fluid model on the expanding domains with general initial data in the framework of weak solutions. We prove rigorously that the weak solutions of the compressible two-fluid model converge to the strong solution of the incompressible Euler equations in the time interval provided that the latter exists and the tool is based on the refined relative entropy method. Moreover, thanks to the Strichartz's estimates of linear wave equations, we also obtain the convergence rates. My talk will be based on a joint work with Professor Young-Sam Kwon.

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"PD-陸冰滢 Bing-Ying Lu" <bylu@gate.sinica.edu.tw>, Sinica

Title: The universality of the semi-classical sine-Gordon equation at the gradient catastrophe

Abstract: We study the semi-classical sine-Gordon equation with pure impulse initial data below the threshold of rotation:

$$\varepsilon^2 u_{tt} - \varepsilon^2 u_{xx} + \sin(u) = 0, u(x, 0) \equiv 0, \varepsilon u_t(x, 0) = G(x) \leq 0, \text{ and } |G(0)| < 2.$$

A dispersive-regularized shock forms in finite time. Using Riemann–Hilbert analysis, we rigorously studied the asymptotics near a certain gradient catastrophe. In accordance with a conjecture made by Dubrovin et. al., the asymptotics in this region is universally (insensitive to initial condition) described by the tritronquée solution to the Painlevé equation. Furthermore, we are able to universally characterize the shapes of the spike-like local structures (rogue wave on periodic background) on top of the poles of the tritronquée solution. (Joint with Peter Miller)

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Applied Math:

"P-王雲哲 Yun-Che Wang" <yunche@mail.ncku.edu.tw>, Civil Engineering, National Cheng Kung University

Title: On the extreme viscoelastic properties in composite materials due to fields governed by Allen-Cahn type PDEs

Abstract: In the framework of the Ginzburg-Landau phase transition theory, ferroelastic solid-solid phase transformations are phenomenologically modeled by the Allen-Cahn-type parabolic partial differential equations that govern the order-parameter fields. In the vicinity of the phase transition, the energy landscape of the system changes from a convex to non-convex profile, hence the interactions between the transforming domains and their surroundings give rise to extreme effective physical properties, such as unbounded viscoelastic modulus and damping. Effective negative stiffness arises in the domains with non-convex energy landscape. In this work, it is shown that our finite-element-based phase-field modeling numerical results are consistent with experimental findings. Effects of microstructure on the extreme properties are to be discussed. In addition, a machine-learning method to numerically solve the Allen-Cahn PDEs, along with viscoelasticity equations, will also be discussed. (Joint work with H.W. Lai and P.C. Cheng)

References

1. M.E. Gurtin, Generalized Ginzburg-Landau and Cahn-Hilliard equations based on a microforce balance, *Physica D* 92, 178-192 (1996)
2. Y.C. Wang, H.W. Lai, M.W. Shen, Effects of cracks on anomalous mechanical behavior and energy dissipation of negative-stiffness plates, *Physica Status Solidi B*, 1800489 (2018)
3. J. Han, A. Jentzen, W. E, Solving high-dimensional partial differential equations using deep learning, *Proceedings of the National Academy of Sciences of the United States of America* 115, 8505-8510 (2018)

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"aP-周鼎羸 Dean Chou" <dean@ncu.edu.tw>, National Central University

Title: Utilising cerebroporomechanics to explore neurological conditions

Abstract: The world population is expected to increase to approximately 11 billion by 2100. The ageing population (aged 60 and over) is projected to exceed the number of children in 2047. This will be a situation without precedent. The number of citizens with disorders of old age like Dementia will rise to 115 million worldwide by 2050. The estimated cost of Dementia will also increase, from \$604 billion in 2010 to \$1,117 billion by 2030. At the same time, medical expertise, evidence-driven policymaking and commissioning of services are increasingly evolving the definitive architecture of comprehensive long-term care to account for these changes.

Technological advances, such as those provided by computational science and biomedical engineering, will allow for an expansion in our ability to model and simulate an almost limitless variety of complex problems that have long defied traditional methods of medical practice. Numerical methods and simulation offer the

prospect of improved clinically relevant predictive information, and of course optimisation, enabling more efficient use of resources for designing treatment protocols, risk assessment and urgently needed management of a long-term care system for a wide spectrum of brain disorders. Within this paradigm, the importance of the relationship of senescence of cerebrospinal fluid transport to dementia in the elderly makes the cerebral environment notably worthy of investigation through numerical and computational modelling.

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"aP-鄧君豪 Chun-Hao Teng" <tengch@nchu.edu.tw>, National Chung Hsing University

Title: High-order numerical methods for partial differential equations on a sphere

Abstract: In this talk, we will present computational approaches for solving partial differential equations on spherical surfaces, based on high-order numerical methods. We will use a model advection equation and the shallow water equations as examples to illustrate the computational framework, including domain decomposition of a spherical surface, governing equations in a general curvilinear coordinate, numerical schemes for simulations, and parallel computational efficiency.

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Plenary Talks:

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"aP-楊劫之 Ryosuke Takahashi" <tryotriple@gmail.com>, National Cheng Kung University

Title: Kapustin-Witten Equation, the Estimate for Yang-Mills Energy and First Pontryagin Number

Abstract: We will briefly introduce some background works on Kapustin-Witten equation and Nahm pole boundary condition. We will sketch the proof of Yang-Mills energy bound for the moduli space of Kapustin-Witten solutions on  $S^3 \times R^+$  with empty knot:

$$\int_M |F_A|^2 < C$$

for any  $(A; \phi) \in \mathfrak{M}$ . We will also prove a formula for the first Pontryagin number for the moduli space of Kapustin-Witten solutions in the general case and propose a way to obtain its bound.

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"PD-李信儀 Hsin-Yi Lee" <apostol2000@hotmail.com>, National Central University

Title: Global Shock Wave Solutions of Hyperbolic Balance Laws for Multi-lane Traffic Flow Model.

Abstract: In this talk, we consider a multi-lanes model of traffic flow, which is governed by a hyperbolic system of balance laws. The system of balance laws is given as a 2 by 2 nonlinear hyperbolic system with



