1. ACD 2. ABD

3. BCD 4. AE

5. ABE 6. BCE

1. 設點 $P(t^2/2, t)$ 在拋物線 $y^2 = 2x$ 上, 定義

$$\sqrt{f(t)} = 距離 = \sqrt{((t^2/2)-1)^2 + (t-1)^2}$$

$$f'(t) = 2((t^2/2)-1)\cdot t + 2(t-1) = t^3 - 2, \quad f''(t) = 3t^2$$

$f'(t) = 0, \Leftrightarrow t = \sqrt[3]{2}$, 且 $f''(\sqrt[3]{2}) > 0$, 極小值。

最短距離爲 $\sqrt{f(\sqrt[3]{2})} = \sqrt{((\sqrt[3]{2}^2/2)-1)^2 + (\sqrt[3]{2}-1)^2}$

2. 令 $f(x) = \tan x$, $f'(x) = \sec^2 x \geq 1$.

By Mean Value Theorem,

$$\tan b - \tan a = f'(c)(b-a)$$

$$\tan b - \tan a \geq (b-a)$$

3. (a)

$$f'(x) = \frac{(1/x)^2}{(1/x)^4+1}\cdot\frac{-1}{x^2} + \frac{1}{x^4+1} = 0$$

(b)

$$\begin{aligned} f(1)+f(-1) &= \int_0^1 \frac{t^2}{t^4+1}dt + \int_0^1 \frac{1}{t^4+1}dt + \int_0^{-1} \frac{t^2}{t^4+1}dt + \int_0^{-1} \frac{1}{t^4+1}dt \\ &= \int_0^1 \frac{t^2}{t^4+1}dt + \int_0^1 \frac{1}{t^4+1}dt - \int_{-1}^0 \frac{t^2}{t^4+1}dt - \int_{-1}^0 \frac{1}{t^4+1}dt \\ &= 0 \end{aligned}$$

(因爲兩個函數都是偶函數)

(c) $f(x)$ 在 $x=0$ 處沒有定義, 又 $f'(0) = 0$. 所以 $f(x)$ 在 x 爲正數時爲一常數函數, $f(x)$ 在 x 爲負數時爲另一個常數函數。

$f(3) = f(1)$, $f(-2) = f(-1)$, $f(3)+f(-2) = f(1)+f(-1) = 0$.

4. (a)

$$f'(x) = \frac{x(1/x) - 1\cdot\ln x}{x^2} = \frac{1-\ln x}{x^2}$$

$f'(e) = 0$. $x = e$ 是唯一的極值, 且 $f'(x) > 0$ if $x < e$, $f'(x) < 0$ if $x > e$,

極大值 $f(e)=1/e$.

(b) 由上題,$f(e)>f(\pi)$,

$$\frac{\ln e}{e}>\frac{\ln\pi}{\pi},\quad \pi\ln e>e\ln\pi,\quad \ln e^{\pi}>\ln\pi^{e}$$

而 $\ln x$ 爲遞增函數。

5.

$$\begin{aligned}\int\sin(\sqrt[3]{x})\,dx &= \int\sin(u)\cdot 3u^2\,du \quad (x=u^3)\\ &= 3u^2(-\cos u)-3\int 2u(-\cos u)\,du\\ &= -3u^2\cos u+6\int u\cos u\,du\\ &= -3u^2\cos u+6\left(u\sin u-\int\sin u\,du\right)\\ &= -3u^2\cos u+6u\sin u+6\cos u+C\\ &= -3(\sqrt[3]{x})^2\cos(\sqrt[3]{x})+6\sqrt[3]{x}\sin(\sqrt[3]{x})+6\cos(\sqrt[3]{x})+C\end{aligned}$$

6.

$$\lim_{x\to0^+}x^{\frac{3}{5}}(\ln x)=\lim_{x\to0^+}\frac{\ln x}{x^{-\frac{3}{5}}}=\lim_{x\to0^+}\frac{x^{-1}}{-\frac{3}{5}x^{-\frac{8}{5}}}=\lim_{x\to0^+}-\frac{5}{3}x^{\frac{3}{5}}=0$$

$$\lim_{x\to0^+}x(\ln x)^{\frac{5}{3}}=[\lim_{x\to0^+}x^{\frac{3}{5}}(\ln x)]^{\frac{5}{3}}=0$$

7.

$$\cos x=1-\frac{x^2}{2!}+\frac{x^4}{4!}-\cdots+(-1)^k\frac{x^{2k}}{(2k)!}+\cdots$$
$$\cos x^2=1-\frac{x^4}{2!}+\frac{x^8}{4!}-\cdots+(-1)^k\frac{x^{4k}}{(2k)!}+\cdots$$

x^{4k} 項才會出現, 2014 不是 4 的倍數, x^{2014} 係數爲 0

Taylor series of $f(x)$ at $x=0$:

$$\sum_{n=0}^{\infty}\frac{f^{(n)}(0)x^n}{n!}$$

所以 $f^{(2014)}(0)=0$.