Hw2 (1.4-11.7) (21893918)

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1. Question Details SCalcET9M 11.4.001. [4783394]

Suppose $\sum a_n$ and $\sum b_n$ are series with positive terms and $\sum b_n$ is known to be convergent.

(a) If $a_n > b_n$ for all n, what, if anything, can you say about $\sum a_n$? Explain.

- ○ $\sum a_n$ converges if and only if $a_n \le 2b_n$.
- ○ $\sum a_n$ converges if and only if $a_n \le 4b_n$.
- ○ We cannot say anything about $\sum a_n$.
- ○ $\sum a_n$ diverges by the Comparison Test.
- ○ $\sum a_n$ converges by the Comparison Test.

(b) If $a_n < b_n$ for all n, what, if anything, can you say about $\sum a_n$? Explain.

- ○ $\sum a_n$ diverges by the Comparison Test.
- ○ $\sum a_n$ converges if and only if $\frac{b_n}{4} \le a_n \le b_n$.
- ○ $\sum a_n$ converges if and only if $\frac{b_n}{2} \le a_n \le b_n$.
- ○ $\sum a_n$ converges by the Comparison Test.
- ○ We cannot say anything about $\sum a_n$.

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2. Question Details SCalcET9M 11.4.002. [4783033]

Suppose $\sum a_n$ and $\sum b_n$ are series with positive terms and $\sum b_n$ is known to be divergent.

(a) If $a_n > b_n$ for all n, what, if anything, can you say about $\sum a_n$? Explain.

- $\sum a_n$ converges if and only if $n \cdot a_n \geq b_n$.
- We cannot say anything about $\sum a_n$.
- $\sum a_n$ diverges by the Comparison Test.
- $\sum a_n$ converges by the Comparison Test.
- $\sum a_n$ converges if and only if $2a_n \geq b_n$.

(b) If $a_n < b_n$ for all n, what, if anything, can you say about $\sum a_n$? Explain.

- $\sum a_n$ diverges by the Comparison Test.
- $\sum a_n$ converges if and only if $a_n \leq \frac{b_n}{n}$.
- $\sum a_n$ converges by the Comparison Test.
- We cannot say anything about $\sum a_n$.
- $\sum a_n$ converges if and only if $a_n \leq \frac{b_n}{4}$.

Need Help? Read It

3. Question Details SCalcET9M 11.4.003. [5094126]

(a) Consider the following statement.

The direct comparison test can be used to show that the first series converges by comparing it to the second series.

$$\sum_{n=2}^{\infty} \frac{n}{n^3+5}, \quad \sum_{n=2}^{\infty} \frac{1}{n^2}$$

The following is a proposed proof for the statement.

1. We have $\frac{n}{n^3+5} < \frac{n}{n^3} = \frac{1}{n^2}$ for all $n \le 2$.
2. The summation $\sum_{n=2}^{\infty} \frac{1}{n^2}$ converges because it is a p-series with $p = 2 > 1$.
3. So $\sum_{n=2}^{\infty} \frac{n}{n^3+5}$ converges by part (i) of the direct comparison test.

Identify the error(s) in the proposed proof. (Select all that apply.)

- ☐ The first sentence should say $\frac{n}{n^3+5} > \frac{n}{n^3} = \frac{1}{n^2}$ instead of $\frac{n}{n^3+5} < \frac{n}{n^3} = \frac{1}{n^2}$.
- ☐ The first sentence should say $n \ge 2$ instead of $n \le 2$.
- ☐ The second sentence states that the summation is a p-series when this is not the case.
- ☐ The second sentence should say diverges instead of converges.
- ☐ The third sentence should say diverges instead of converges.

(b) Consider the following statement.

The limit comparison test can be used to show that the first series converges by comparing it to the second series.

$$\sum_{n=2}^{\infty} \frac{n}{n^3-5}, \quad \sum_{n=2}^{\infty} \frac{1}{n^2}$$

The following is a proposed proof for the statement.

1. Use the limit comparison test with $a_n = \frac{n}{n^3-5}$ and $b_n = \frac{n^2}{1}$.
2. We take $\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{n}{n^3-5} \cdot \frac{n^2}{1} = \lim_{n\to\infty} \frac{n^3}{n^3\left(1-\frac{5}{n^3}\right)}$
$= \lim_{n\to\infty} \frac{1}{1-\frac{5}{n^3}} = -1 < 0.$
3. Since $\sum_{n=2}^{\infty} \frac{1}{n^2}$ is a convergent (partial) p-series [$p = 2 > 1$], the series $\sum_{n=2}^{\infty} \frac{n}{n^3-5}$ converges.

Identify the error(s) in the proposed proof. (Select all that apply.)

- ☐ The first sentence should say $a_n = \frac{n}{n^3+5}$ instead of $a_n = \frac{1}{n^3-5}$.
- ☐ The first sentence should say $b_n = \frac{1}{n^2}$ instead of $b_n = \frac{n^2}{1}$.
- ☐ The second sentence should say $n \to -\infty$ instead of $n \to \infty$.
- ☐ The second sentence should conclude with the statement $= 1 > 0$ instead of $= -1 < 0$.

☐ The third sentence claims that $\sum_{n=2}^{\infty} \frac{1}{n^2}$ is convergent when it is really divergent.

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4. Question Details SCalcET9M 11.4.017. [5094044]

Determine whether the series converges or diverges.

$$\sum_{n=1}^{\infty} \frac{1 + \sin(n)}{e^n}$$

- ○ converges
- ○ diverges

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5. Question Details SCalcET9M 11.4.026.EP. [5093354]

Determine whether the series converges or diverges.

$$\sum_{n=2}^{\infty} \frac{n + 5}{(n + 7)^4}$$

- ○ The series converges by the Direct Comparison Test. Each term is less than that of the harmonic series.
- ○ The series converges by the Limit Comparison Test with a convergent p-series.
- ○ The series diverges by the Limit Comparison Test with a divergent p-series.
- ○ The series diverges by the Direct Comparison Test. Each term is greater than that of a divergent geometric series.

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6. Question Details SCalcET9M 11.5.019. [5093725]

Test the series for convergence or divergence using the Alternating Series Test.

$$\sum_{n=1}^{\infty} (-1)^n \frac{n^2}{7^n}$$

Identify b_n.

Evaluate the following limit.

$$\lim_{n\to\infty} b_n$$

Since $\lim_{n\to\infty} b_n$ [?] 0 and b_{n+1} [?] b_n for all $n > 2$, [---Select---] .

Need Help? Read It

7. Question Details SCalcET9M 11.5.021. [5093781]

(a) What does it mean for a series to be absolutely convergent?

- ○ $\sum a_n$ is absolutely convergent if $\sum a_n$ diverges and $\sum |a_n|$ diverges.
- ○ $\sum a_n$ is absolutely convergent if $\sum a_n$ converges and $\sum |a_n|$ converges.
- ○ $\sum a_n$ is absolutely convergent if $\sum a_n$ diverges but $\sum |a_n|$ converges.
- ○ $\sum a_n$ is absolutely convergent if $\sum a_n$ converges but $\sum |a_n|$ diverges.

(b) What does it mean for a series to be conditionally convergent?

- ○ $\sum a_n$ is absolutely convergent if $\sum a_n$ diverges and $\sum |a_n|$ diverges.
- ○ $\sum a_n$ is conditionally convergent if $\sum a_n$ converges and $\sum |a_n|$ converges.
- ○ $\sum a_n$ is conditionally convergent if $\sum a_n$ converges but $\sum |a_n|$ diverges.
- ○ $\sum a_n$ is absolutely convergent if $\sum a_n$ diverges but $\sum |a_n|$ converges.

(c) If the series of positive terms $\sum_{n=1}^{\infty} b_n$ converges, then what can you say about the series $\sum_{n=1}^{\infty} (-1)^n b_n$?

- ○ $\sum_{n=1}^{\infty} (-1)^n b_n$ is conditionally convergent.
- ○ $\sum_{n=1}^{\infty} (-1)^n b_n$ is absolutely convergent.
- ○ More information is needed to determine if $\sum_{n=1}^{\infty} (-1)^n b_n$ converges or diverges.
- ○ $\sum_{n=1}^{\infty} (-1)^n b_n$ converges, but more information is needed to determine if it is absolutely or conditionally convergent.

Need Help? Read It

8. Question Details SCalcET9M 11.5.023. [5093418]

Determine whether the series is absolutely convergent, conditionally convergent, or divergent.

$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{\sqrt[6]{n^5}}$$

- ○ absolutely convergent
- ○ conditionally convergent
- ○ divergent

Need Help? Read It

9. Question Details SCalcET9M 11.6.003. [4782593]

Use the Ratio Test to determine whether the series is convergent or divergent.

$$\sum_{n=1}^{\infty} \frac{n}{6^n}$$

Identify a_n.

Evaluate the following limit.

$$\lim_{n \to \infty} \left| \frac{a_{n+1}}{a_n} \right|$$

Since $\lim_{n \to \infty} \left| \frac{a_{n+1}}{a_n} \right|$? 1, ---Select---.

Need Help? Read It Watch It

10. Question Details SCalcET9M 11.6.011. [5094090]

Use the Ratio Test to determine whether the series is convergent or divergent.

$$\sum_{n=1}^{\infty} \frac{n\pi^n}{(-7)^{n-1}}$$

Identify a_n.

Evaluate the following limit.

$$\lim_{n \to \infty} \left| \frac{a_{n+1}}{a_n} \right|$$

Since $\lim_{n \to \infty} \left| \frac{a_{n+1}}{a_n} \right|$? 1, ---Select---.

Need Help? Read It

11. Question Details SCalcET9M 11.6.013. [4783233]

Use the Ratio Test to determine whether the series is convergent or divergent.

$$\sum_{n=1}^{\infty} \frac{\cos\left(\frac{n\pi}{6}\right)}{n!}$$

Identify a_n.

Evaluate the following limit.

$$\lim_{n \to \infty} \left| \frac{a_{n+1}}{a_n} \right|$$

Since $\lim_{n \to \infty} \left| \frac{a_{n+1}}{a_n} \right|$? 1, ---Select--- .

Need Help? Read It Watch It

12. Question Details SCalcET9M 11.6.039. [4782963]

Determine whether the Ratio Test is inconclusive (that is, it fails to give a definite answer), conclusive (convergent), or conclusive (divergent) for each series.

(a) $\sum_{n=2}^{\infty} \frac{5}{n^3}$

- inconclusive
- conclusive (convergent)
- conclusive (divergent)

(b) $\sum_{n=1}^{\infty} \frac{n}{4^n}$

- inconclusive
- conclusive (convergent)
- conclusive (divergent)

(c) $\sum_{n=2}^{\infty} \frac{(-5)^{n-1}}{\sqrt{n}}$

- inconclusive
- conclusive (convergent)
- conclusive (divergent)

(d) $\sum_{n=2}^{\infty} \frac{\sqrt{n}}{1+n^2}$

- inconclusive
- conclusive (convergent)
- conclusive (divergent)

Need Help? Read It Watch It

13. Question Details SCalcET9M 11.4.037.MI. [4782552]

Determine whether the series converges or diverges.

$$\sum_{n=1}^{\infty} 4\sin\left(\frac{3}{n}\right)$$

- ○ convergent
- ○ divergent

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14. Question Details SCalcET9M 11.7.007. [5093594]

Two similar-looking series are given. Test each one for convergence or divergence.

(a) $\sum_{n=1}^{\infty} \frac{1}{n + n!}$

- ○ convergent
- ○ divergent

(b) $\sum_{n=1}^{\infty} \left(\frac{1}{n} + \frac{1}{n!}\right)$

- ○ convergent
- ○ divergent

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15. Question Details SCalcET9M 11.7.013. [4782639]

Test the series for convergence or divergence.

$$\sum_{n=1}^{\infty} \frac{e^n}{n^2}$$

- ○ convergent
- ○ divergent

Need Help? Read It

16. Question Details SCalcET9M 11.7.018. [4782845]

Test the series for convergence or divergence.

$$\sum_{n=1}^{\infty} n^4 e^{-n^5}$$

- ○ convergent
- ○ divergent

Need Help? Read It

17. Question Details SCalcET9M 11.7.025. [4782409]

Test the series for convergence or divergence.

$$\sum_{n=1}^{\infty} \frac{7 \cdot 15 \cdot 23 \cdot \cdots (8n-1)}{8 \cdot 17 \cdot 26 \cdot \cdots (9n-1)}$$

- ○ convergent
- ○ divergent

Need Help? Read It Watch It

18. Question Details SCalcET9M 11.7.001. [5094019]

Two similar-looking series are given. Test each one for convergence or divergence.

(a) $\sum_{n=1}^{\infty} \frac{1}{6^n}$

- ○ convergent
- ○ divergent

(b) $\sum_{n=1}^{\infty} \frac{1}{6^n + n}$

- ○ convergent
- ○ divergent

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19. Question Details SCalcET9M 11.7.003. [5093367]

Two similar-looking series are given. Test each one for convergence or divergence.

(a) $\sum_{n=1}^{\infty} \frac{n}{5^n}$

- ○ convergent
- ○ divergent

(b) $\sum_{n=1}^{\infty} \frac{5^n}{n}$

- ○ convergent
- ○ divergent

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20. Question Details SCalcET9M 11.7.009. [4782748]

Test the series for convergence or divergence.

$$\sum_{n=1}^{\infty} \frac{n^4 - 1}{n^5 + 1}$$

- ○ convergent
- ○ divergent

Need Help? Read It

21. Question Details SCalcET9M 11.7.019. [4783305]

Test the series for convergence or divergence.

$$\sum_{n=1}^{\infty} \left(\frac{1}{n^5} + \frac{1}{5^n} \right)$$

- ○ convergent
- ○ divergent

Need Help? Read It Watch It

22. Question Details SCalcET9M 11.7.022. [4783218]

Test the series for convergence or divergence.

$$\sum_{n=1}^{\infty} \frac{\sin(5n)}{1 + 5^n}$$

- ○ convergent
- ○ divergent

Need Help? Read It

23. Question Details SCalcET9M 11.7.023. [4782830]

Test the series for convergence or divergence.

$$\sum_{k=1}^{\infty} \frac{9^{k-1} 8^{k+1}}{k^k}$$

- ○ convergent
- ○ divergent

Need Help? Read It Watch It

24. Question Details SCalcET9M 11.7.033. [5093703]

Test the series for convergence or divergence.

$$\sum_{n=1}^{\infty} \frac{5 - \cos(n)}{\sqrt{n}}$$

- ○ convergent
- ○ divergent

Need Help? Read It

25. Question Details SCalcET9M 11.7.037. [4782835]

Test the series for convergence or divergence.

$$\sum_{k=1}^{\infty} \frac{k \ln(k)}{(k+4)^3}$$

- ○ convergent
- ○ divergent

Need Help? Read It Watch It

26. Question Details SCalcET9M 11.5.029. [5093445]

Determine whether the series is absolutely convergent, conditionally convergent, or divergent.

$$\sum_{n=1}^{\infty} \frac{5 + 5\sin(n)}{n^9}$$

- ○ absolutely convergent
- ○ conditionally convergent
- ○ divergent

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27. Question Details SCalcET9M 11.5.031. [4783106]

Use any test to determine whether the series is absolutely convergent, conditionally convergent, or divergent.

$$\sum_{n=2}^{\infty} \frac{(-1)^n}{\ln(5n)}$$

- ○ absolutely convergent
- ○ conditionally convergent
- ○ divergent

Need Help? Read It Watch It

28. Question Details SCalcET9M 11.5.033. [5093353]

Determine whether the series is absolutely convergent, conditionally convergent, or divergent.

$$\sum_{n=1}^{\infty} \frac{\cos(n\pi)}{5n+3}$$

- ○ absolutely convergent
- ○ conditionally convergent
- ○ divergent

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Assignment Details