Hw 5 (12.1-12.5) (22002973)

Question 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35

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1. Question Details SCalcET9M 12.1.010. [5093663]

Find the distance between the given points.

$(-1, -5, 2)$, $(6, 4, 3)$

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2. Question Details SCalcET9M 12.1.017.MI. [4783509]

Find an equation of the sphere that passes through the point $(8, 3, -3)$ and has center $(5, 6, 3)$.

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3. Question Details SCalcET9M 12.1.049. [4783909]

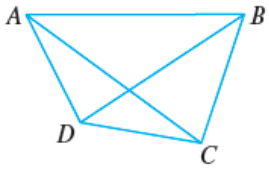
Find an equation of the set of all points equidistant from the points $A(-3, 5, 3)$ and $B(6, 2, -2)$.

Describe the set.

- ○ a sphere with diameter AB
- ○ a cube with diagonal AB
- ○ a plane perpendicular to AB
- ○ a line perpendicular to AB

4. Question Details SCalcET9M 12.2.004. [4784333]

Write each combination of vectors as a single vector.



(a) $\overrightarrow{AB} + \overrightarrow{BC}$

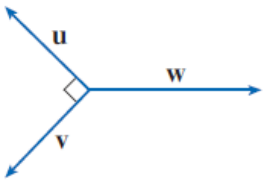
(b) $\overrightarrow{CD} + \overrightarrow{DB}$

(c) $\overrightarrow{DB} - \overrightarrow{AB}$

(d) $\overrightarrow{DC} + \overrightarrow{CA} + \overrightarrow{AB}$

5. Question Details SCalcET9M 12.2.008. [5093736]

If the vectors in the figure satisfy $|\mathbf{u}| = |\mathbf{v}| = 1$ and $\mathbf{u} + \mathbf{v} + \mathbf{w} = \mathbf{0}$, what is $|\mathbf{w}|$?



$|\boldsymbol{w}| =$

6. Question Details SCalcET9M 12.2.020. [5093286]

Find $\mathbf{a} + \mathbf{b}$, $4\mathbf{a} + 2\mathbf{b}$, $|\mathbf{a}|$, and $|\mathbf{a} - \mathbf{b}|$. (Simplify your vectors completely.)

$\mathbf{a} = 4\mathbf{i} + \mathbf{j}, \quad \mathbf{b} = \mathbf{i} - 3\mathbf{j}$

$\mathbf{a} + \mathbf{b} =$

$4\mathbf{a} + 2\mathbf{b} =$

$|\mathbf{a}| =$

$|\mathbf{a} - \mathbf{b}| =$

7. Question Details SCalcET9M 12.2.026. [4783679]

Find the vector that has the same direction as $\langle 2, 6, -9 \rangle$ but has length 4.

8. Question Details SCalcET9M 12.3.003. [5094122]

Find $\mathbf{a} \cdot \mathbf{b}$.

$\mathbf{a} = \langle 4.5, 0.2 \rangle, \quad \mathbf{b} = \langle -5, 2 \rangle$

9. Question Details SCalcET9M 12.3.009. [4784395]

Find $\mathbf{a} \cdot \mathbf{b}$.

$|\mathbf{a}| = 7, \quad |\mathbf{b}| = 4$, the angle between $\mathbf{a}$ and $\mathbf{b}$ is 30°.

10. Question Details SCalcET9M 12.3.015. [4784028]

Find the angle between the vectors. (First find an exact expression and then approximate to the nearest degree.)

$\mathbf{a} = \langle 7, 2 \rangle, \quad \mathbf{b} = \langle 2, -1 \rangle$

exact

approximate °

11. Question Details SCalcET9M 12.3.023. [4784439]

Determine whether the given vectors are orthogonal, parallel, or neither.

(a) $\mathbf{a} = \langle 9, 3 \rangle$, $\mathbf{b} = \langle -2, 6 \rangle$

- ○ orthogonal
- ○ parallel
- ○ neither

(b) $\mathbf{a} = \langle 6, 7, -3 \rangle$, $\mathbf{b} = \langle 5, -1, 7 \rangle$

- ○ orthogonal
- ○ parallel
- ○ neither

(c) $\mathbf{a} = -4\mathbf{i} + 8\mathbf{j} + 12\mathbf{k}$, $\mathbf{b} = 3\mathbf{i} - 6\mathbf{j} - 9\mathbf{k}$

- ○ orthogonal
- ○ parallel
- ○ neither

(d) $\mathbf{a} = 4\mathbf{i} - \mathbf{j} + 4\mathbf{k}$, $\mathbf{b} = 5\mathbf{i} + 12\mathbf{j} - 2\mathbf{k}$

- ○ orthogonal
- ○ parallel
- ○ neither

12. Question Details SCalcET9M 12.3.025.EP. [5093932]

Consider the triangle with vertices $P(2, -5, -3)$, $Q(3, -2, -5)$, and $R(7, -4, -6)$.

Determine the following vectors.

$\overrightarrow{QP}$ =

$\overrightarrow{QR}$ =

Find $\overrightarrow{QP} \cdot \overrightarrow{QR}$.

$\overrightarrow{QP} \cdot \overrightarrow{QR}$ =

Is the given triangle right-angled?

- ○ Yes, it is right-angled.
- ○ No, it is not right-angled.

13. Question Details SCalcET9M 12.3.039. [4783919]

Find the scalar and vector projections of **b** onto **a**.

$\mathbf{a} = \langle -3, 4 \rangle$, $\mathbf{b} = \langle 3, 6 \rangle$

scalar projection of **b** onto **a**

vector projection of **b** onto **a**

14. Question Details SCalcET9M 12.3.044. [5093310]

Find the scalar and vector projections of **b** onto **a**.

$\mathbf{a} = \mathbf{i} + 4\mathbf{j} + 5\mathbf{k}, \quad \mathbf{b} = 6\mathbf{i} - \mathbf{k}$

scalar projection of **b** onto **a**

vector projection of **b** onto **a**

15. Question Details SCalcET9M 12.3.AE.003. [4784377]

Video Example

EXAMPLE 3 Find the angle between the vectors $\mathbf{a} = \langle 4, 4, -2 \rangle$ and $\mathbf{b} = \langle 3, -1, 3 \rangle$.

SOLUTION Since

$$|\mathbf{a}| = \sqrt{4^2 + 4^2 + (-2)^2} =$$

and

$$|\mathbf{b}| = \sqrt{3^2 + (-1)^2 + 3^2} =$$

and since

$$\mathbf{a} \cdot \mathbf{b} = (4)(\quad) + (4)(-1) + (-2)(\quad)$$
$$=$$

we have from this corollary

$$\cos(\theta) = \frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}||\mathbf{b}|} =$$

So the angle between **a** and **b** is as follows. (Round your final answer to two decimal places.)

$$\theta = \cos^{-1}(\quad) \approx \text{ rad}$$

16. Question Details SCalcET9M 12.4.002. [5093369]

Find the cross product $\mathbf{a} \times \mathbf{b}$.

$\mathbf{a} = \langle 7, 6, -5 \rangle, \quad \mathbf{b} = \langle 2, -1, 1 \rangle$

Verify that it is orthogonal to both **a** and **b**.

$(\mathbf{a} \times \mathbf{b}) \cdot \mathbf{a} =$

$(\mathbf{a} \times \mathbf{b}) \cdot \mathbf{b} =$

17. Question Details SCalcET9M 12.4.013. [4784380]

State whether each expression is meaningful. If not, explain why. If so, state whether it is a vector or a scalar.

(a) $\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})$

- ○ The expression is meaningful. It is a vector.
- ○ The expression is meaningful. It is a scalar.
- ○ The expression is meaningless. The cross product is defined only for two vectors.
- ○ The expression is meaningless. The dot product is defined only for two vectors.

(b) $\mathbf{a} \times (\mathbf{b} \cdot \mathbf{c})$

- ○ The expression is meaningful. It is a vector.
- ○ The expression is meaningful. It is a scalar.
- ○ The expression is meaningless. The cross product is defined only for two vectors.
- ○ The expression is meaningless. The dot product is defined only for two vectors.

(c) $\mathbf{a} \times (\mathbf{b} \times \mathbf{c})$

- ○ The expression is meaningful. It is a vector.
- ○ The expression is meaningful. It is a scalar.
- ○ The expression is meaningless. The cross product is defined only for two vectors.
- ○ The expression is meaningless. The dot product is defined only for two vectors.

(d) $\mathbf{a} \cdot (\mathbf{b} \cdot \mathbf{c})$

- ○ The expression is meaningful. It is a vector.
- ○ The expression is meaningful. It is a scalar.
- ○ The expression is meaningless. The cross product is defined only for two vectors.
- ○ The expression is meaningless. The dot product is defined only for two vectors.

(e) $(\mathbf{a} \cdot \mathbf{b}) \times (\mathbf{c} \cdot \mathbf{d})$

- ○ The expression is meaningful. It is a vector.
- ○ The expression is meaningful. It is a scalar.
- ○ The expression is meaningless. The cross product is defined only for two vectors.
- ○ The expression is meaningless. The dot product is defined only for two vectors.

(f) $(\mathbf{a} \times \mathbf{b}) \cdot (\mathbf{c} \times \mathbf{d})$

- ○ The expression is meaningful. It is a vector.
- ○ The expression is meaningful. It is a scalar.
- ○ The expression is meaningless. The cross product is defined only for two vectors.
- ○ The expression is meaningless. The dot product is defined only for two vectors.

18. Question Details SCalcET9M 12.4.023. [4784130]

If **a**, **b**, and **c** are vectors and c is a scalar, then we have the following properties.

1. $\mathbf{a} \times \mathbf{b} = -\mathbf{b} \times \mathbf{a}$
2. $(c\mathbf{a}) \times \mathbf{b} = c(\mathbf{a} \times \mathbf{b}) = \mathbf{a} \times (c\mathbf{b})$
3. $\mathbf{a} \times (\mathbf{b} + \mathbf{c}) = \mathbf{a} \times \mathbf{b} + \mathbf{a} \times \mathbf{c}$
4. $(\mathbf{a} + \mathbf{b}) \times \mathbf{c} = \mathbf{a} \times \mathbf{c} + \mathbf{b} \times \mathbf{c}$
5. $\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c}) = (\mathbf{a} \times \mathbf{b}) \cdot \mathbf{c}$
6. $\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = (\mathbf{a} \cdot \mathbf{c})\mathbf{b} - (\mathbf{a} \cdot \mathbf{b})\mathbf{c}$

Prove the property $\mathbf{a} \times \mathbf{b} = -\mathbf{b} \times \mathbf{a}$ of the theorem above.

Let $\mathbf{a} = \langle a_1, a_2, a_3 \rangle$ and $\mathbf{b} = \langle b_1, b_2, b_3 \rangle$. Then,

$\mathbf{a} \times \mathbf{b} =$ [ ]

$= (-1)$ [ ]

$= -\mathbf{b} \times \mathbf{a}$.

19. Question Details SCalcET9M 12.4.028. [5093301]

Find the area of the parallelogram with vertices $P(1, 1, 2)$, $Q(3, 3, 3)$, $R(7, 8, 12)$, and $S(5, 6, 11)$.

[ ]

20. Question Details SCalcET9M 12.4.032. [5093332]

Consider the points below.

$P(3, -2, 5)$, $Q(-1, -2, 3)$, $R(2, 1, -3)$

(a) Find a nonzero vector orthogonal to the plane through the points P, Q, and R.

[ ]

(b) Find the area of the triangle PQR.

[ ]

21. Question Details SCalcET9M 12.4.033. [4784456]

Find the volume of the parallelepiped determined by the vectors **a**, **b**, and **c**.

$\mathbf{a} = \langle 1, 4, 4 \rangle$, $\mathbf{b} = \langle -1, 1, 5 \rangle$, $\mathbf{c} = \langle 5, 1, 3 \rangle$

[ ] cubic units

22. Question Details SCalcET9M 12.4.037. [4784252]

Use the scalar triple product to determine if the vectors $\mathbf{u} = \mathbf{i} + 4\mathbf{j} - 3\mathbf{k}$, $\mathbf{v} = 4\mathbf{i} - \mathbf{j}$, and $\mathbf{w} = 7\mathbf{i} + 11\mathbf{j} - 9\mathbf{k}$ are coplanar.

- ○ Yes, they are coplanar.
- ○ No, they are not coplanar.

23. Question Details SCalcET9M 12.4.038.EP. [5093592]

Consider the following points.

$A(3, 2, 3)$, $B(6, -3, 7)$, $C(9, 0, 0)$, $D(6, 5, -4)$

Let $\mathbf{u} = \overrightarrow{AB}$, $\mathbf{v} = \overrightarrow{AC}$, and $\mathbf{w} = \overrightarrow{AD}$. Find each of these vectors.

$\mathbf{u}$ =

$\mathbf{v}$ =

$\mathbf{w}$ =

Find the scalar triple product $\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w})$.

$\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w})$ =

Do the given points A, B, C, and D all lie in the same plane?

- ○ Yes, they lie in the same plane.
- ○ No, they do not lie in the same plane.

24. Question Details SCalcET9M 12.5.002. [4784296]

Find a vector equation and parametric equations for the line. (Use the parameter t.)

The line through the point $(3, 2.5, 3.6)$ and parallel to the vector $3\mathbf{i} + 3\mathbf{j} - \mathbf{k}$

$\mathbf{r}(t)$ =

$(x(t), y(t), z(t))$ = ( )

25. Question Details SCalcET9M 12.5.006. [4783806]

Find parametric equations for the line. (Use the parameter t.)

the line through the points $\left(0, \frac{1}{2}, 1\right)$ and $(5, 1, -5)$

$(x(t), y(t), z(t))$ = ( )

Find the symmetric equations.

- ○ $x - 5 = 2y - 2 = z + 5$
- ○ $\frac{x + 5}{-6} = 2y - 2 = \frac{z - 5}{5}$
- ○ $2x - 2 = \frac{y - 5}{5} = \frac{z + 5}{-6}$
- ○ $5 + 5x = 1 + \frac{y}{2} = -5 - 6z$
- ○ $\frac{x - 5}{5} = 2y - 2 = \frac{z + 5}{-6}$

26. Question Details SCalcET9M 12.5.016. [4784621]

(a) Find parametric equations for the line through (3, 3, 8) that is perpendicular to the plane $x - y + 4z = 4$. (Use the parameter t.)

$(x(t), y(t), z(t)) = ($ $)$

(b) In what points does this line intersect the coordinate planes?

xy-plane $(x(t), y(t), z(t)) = ($ $)$

yz-plane $(x(t), y(t), z(t)) = ($ $)$

xz-plane $(x(t), y(t), z(t)) = ($ $)$

27. Question Details SCalcET9M 12.5.025. [5093711]

Find an equation of the plane.

the plane through the point (4, −2, 5) and perpendicular to the vector $-\mathbf{i} + 4\mathbf{j} + 5\mathbf{k}$

28. Question Details SCalcET9M 12.5.030. [5093987]

Find an equation of the plane.

the plane that contains the line $x = 2 + t$, $y = 3 - t$, $z = 3 - 3t$ and is parallel to the plane $5x + 2y + z = 8$

29. Question Details SCalcET9M 12.5.032. [5093482]

Find an equation of the plane.

the plane through the origin and the points (5, −3, 2) and (1, 1, 1)

30. Question Details SCalcET9M 12.5.037. [5093643]

Find an equation of the plane.

the plane that passes through the point (4, 3, 2) and contains the line of intersection of the planes $x + 2y + 3z = 1$ and $2x - y + z = -3$

31. Question Details SCalcET9M 12.5.040. [4783889]

Find an equation of the plane.

the plane that passes through the line of intersection of the planes $x - z = 2$ and $y + 3z = 1$ and is perpendicular to the plane $x + y - 4z = 3$

32. Question Details SCalcET9M 12.5.050. [4783772]

Find the cosine of the angle between the planes $x + y + z = 0$ and $x + 3y + 2z = 5$.

33. Question Details SCalcET9M 12.5.059. [4784042]

Find symmetric equations for the line of intersection of the planes.

$3x - 3y - 3z = -6, \quad 3x + y + z = 6$

- $y = 3, \dfrac{x-1}{-12} = \dfrac{z}{12}$
- $x = -1, y + 3 = -z$
- $x = 1, y + 3 = -z$
- $x - 1 = \dfrac{y-3}{-12} = \dfrac{z}{12}$
- $x = 1, y - 3 = -z$

34. Question Details SCalcET9M 12.5.065. [4784399]

Find parametric equations for the line through the point $(0, 2, 2)$ that is parallel to the plane $x + y + z = 1$ and perpendicular to the line $x = 1 + t, y = 2 - t, z = 2t$. (Use the parameter t.)

$(x(t), y(t), z(t)) = ($ $)$

35. Question Details SCalcET9M 12.5.078. [4784173]

Find the distance between the skew lines with parametric equations $x = 1 + t, y = 1 + 6t, z = 2t$, and $x = 1 + 2s, y = 6 + 14s, z = -2 + 5s$.

Assignment Details