Hw 6 (13.1-13.4) (22010378)

Question 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32

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1. Question Details SCalcET9M 13.1.001. [5093346]

Find the domain of the vector function. (Enter your answer using interval notation.)

$$\mathbf{r}(t) = \left\langle \ln(t+4), \frac{t}{\sqrt{25-t^2}}, 2^t \right\rangle$$

---

2. Question Details SCalcET9M 13.1.002. [5093557]

Find the domain of the vector function. (Enter your answer using interval notation.)

$$\mathbf{r}(t) = \cos(t)\,\mathbf{i} + \ln(t)\,\mathbf{j} + \frac{1}{t-9}\,\mathbf{k}$$

---

3. Question Details SCalcET9M 13.1.005. [5093348]

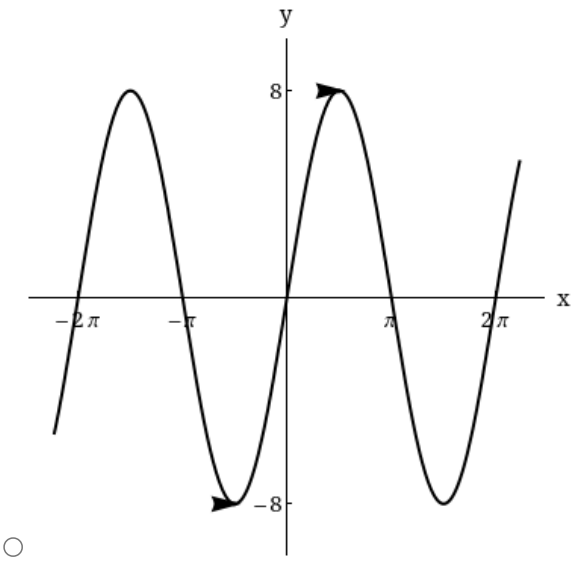
Find the limit.

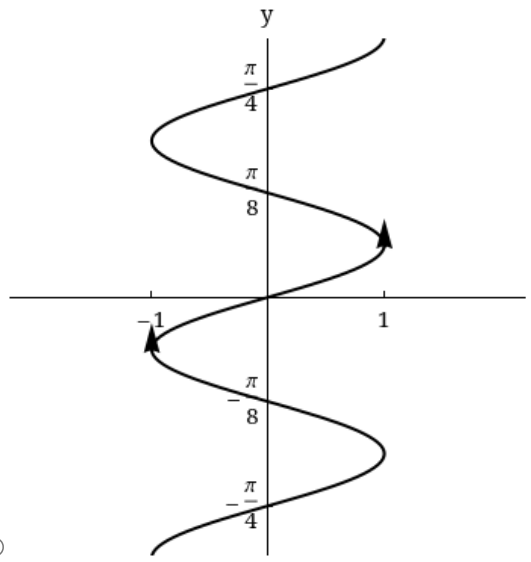
$$\lim_{t\to\infty} \left\langle \frac{7+t^2}{7-t^2}, 7\tan^{-1}(t), \frac{7-e^{-2t}}{t} \right\rangle$$

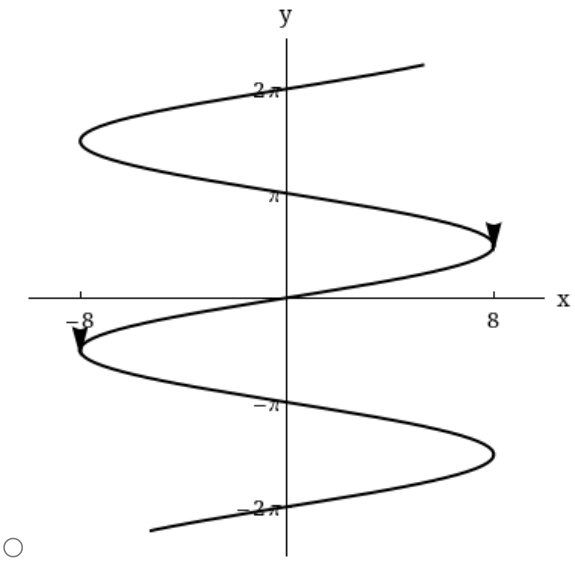
4. Question Details SCalcET9M 13.1.007. [4783925]

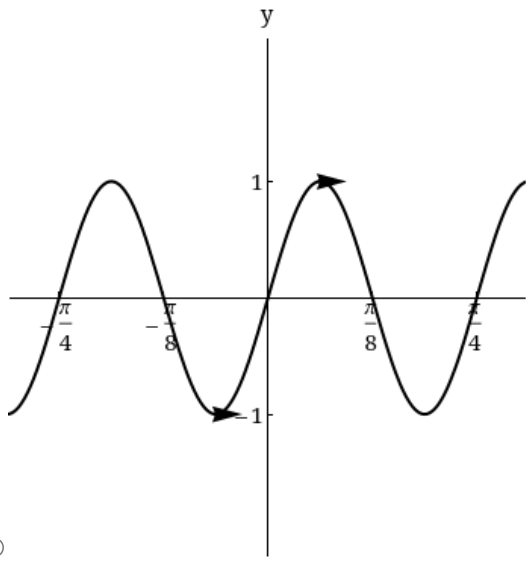
Sketch the curve with the given vector equation. Indicate with an arrow the direction in which t increases.

$\mathbf{r}(t) = \langle \sin(8t), t \rangle$





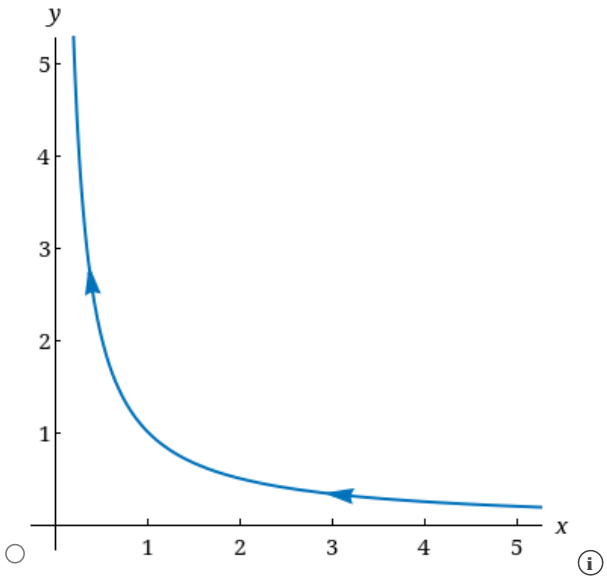


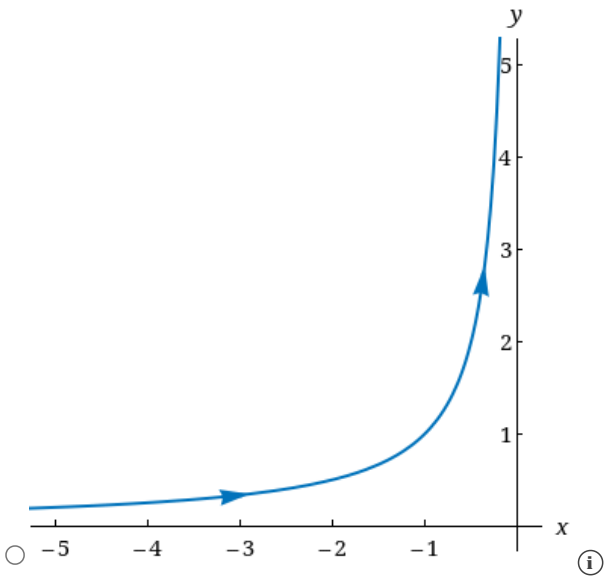


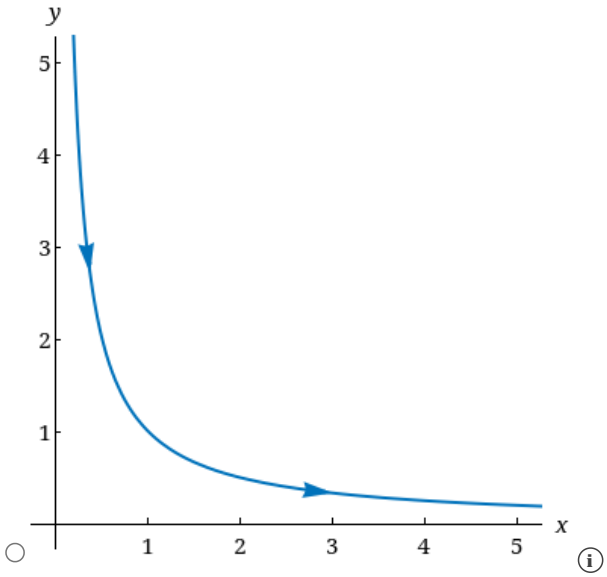
5. Question Details SCalcET9M 13.1.010. [5093904]

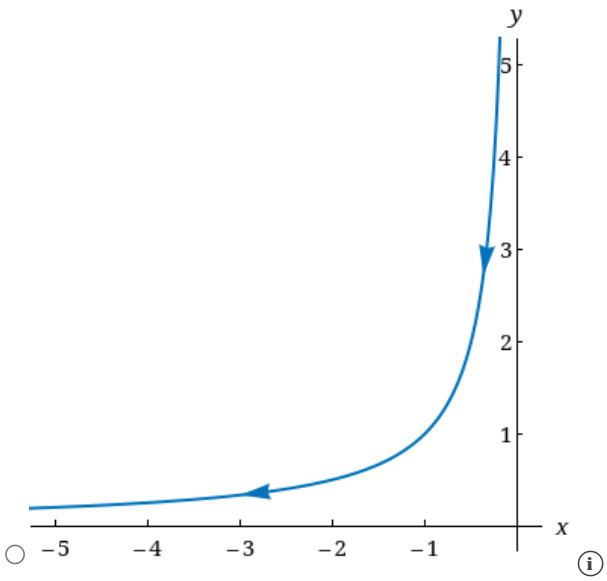
Sketch the curve with the given vector equation. Indicate with an arrow the direction in which t increases.

$$\mathbf{r}(t) = e^{-t}\mathbf{i} + e^{t}\mathbf{j}$$





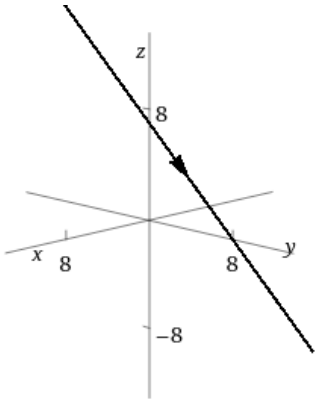

y
5
4
3
2
1
x
1
2
3
4
5

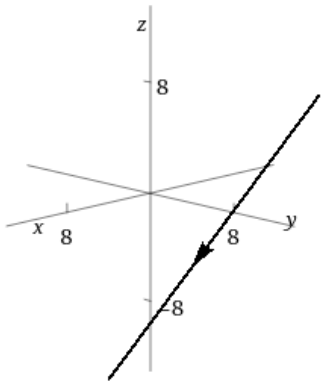

y
5
4
3
2
1
x
−5
−4
−3
−2
−1

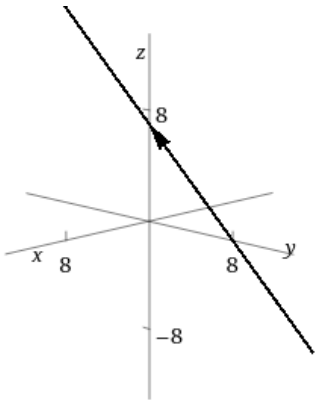
6. Question Details SCalcET9M 13.1.011. [4783777]

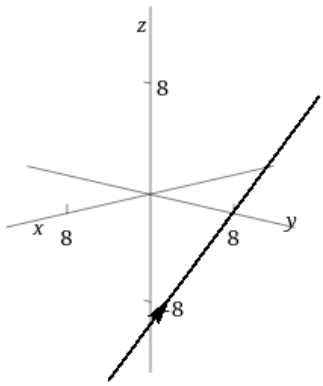
Sketch the curve with the given vector equation. Indicate with an arrow the direction in which t increases.

$\mathbf{r}(t) = \langle t, 8 - t, 2t \rangle$





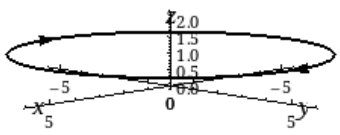


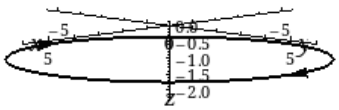


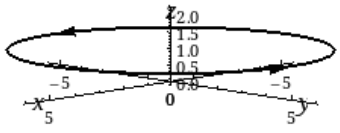
7. Question Details SCalcET9M 13.1.014. [4784511]

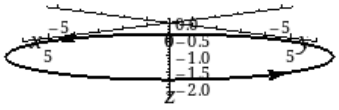
Sketch the curve with the given vector equation. Indicate with an arrow the direction in which t increases.

$\mathbf{r}(t) = 5\cos(t)\mathbf{i} + 5\sin(t)\mathbf{j} + \mathbf{k}$









8. Question Details SCalcET9M 13.1.023. [4783674]

Find a vector equation and parametric equations for the line segment that joins P to Q.

$P(0, -4, 1)$, $Q\left(\frac{1}{2}, \frac{1}{3}, \frac{1}{4}\right)$

vector equation $\mathbf{r}(t) =$

parametric equations $(x(t), y(t), z(t)) =$ ( )

9. Question Details SCalcET9M 13.2.003. [4783753]

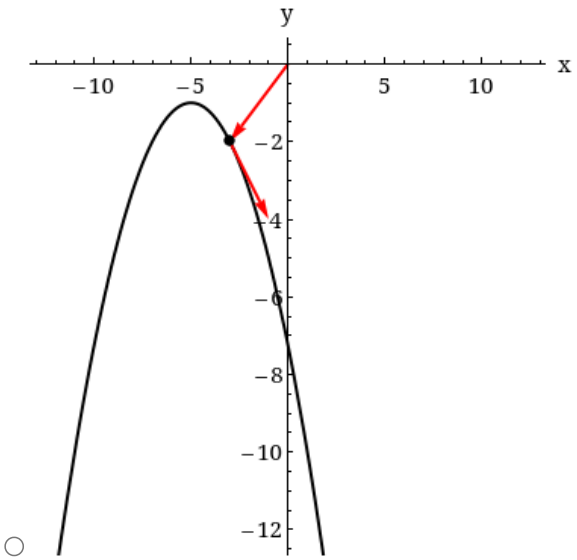
Consider the given vector equation.

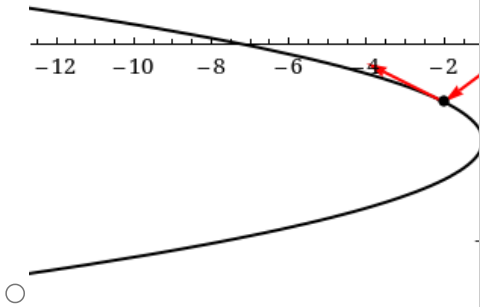
$\mathbf{r}(t) = \langle 2t - 5, t^2 + 1 \rangle$

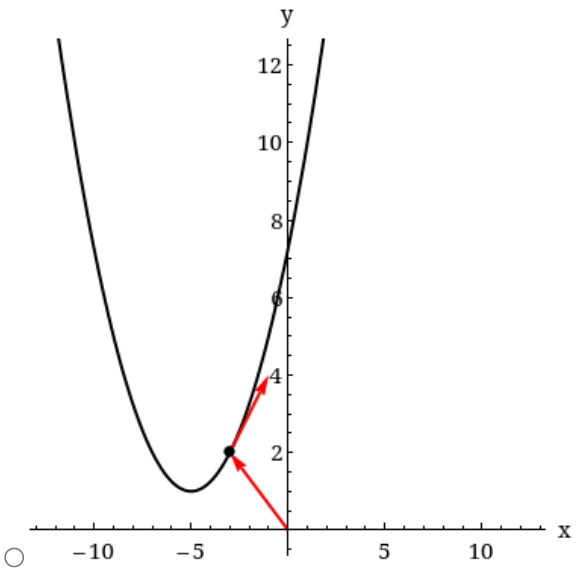
(a) Find $\mathbf{r}'(t)$.

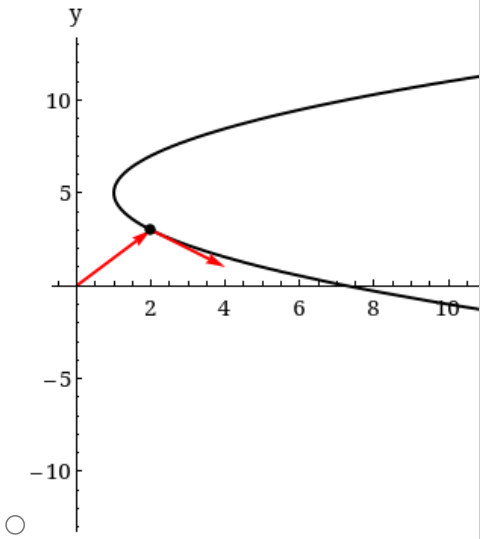
$\mathbf{r}'(t) =$

(b) Sketch the plane curve together with the position vector $\mathbf{r}(t)$ and the tangent vector $\mathbf{r}'(t)$ for the given value of $t = 1$.









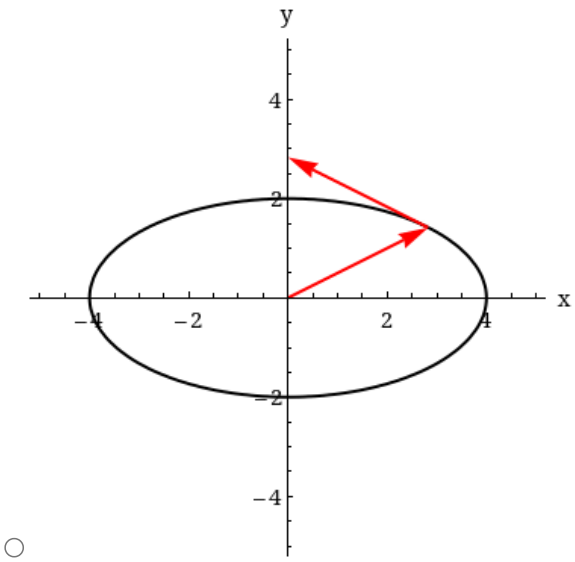
10. Question Details SCalcET9M 13.2.007. [4784625]

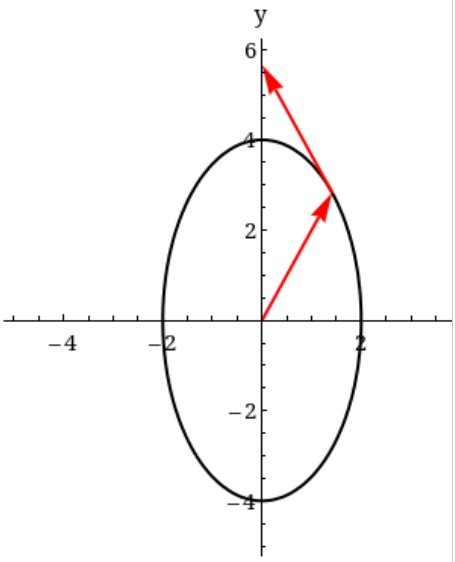
Consider the given vector equation.

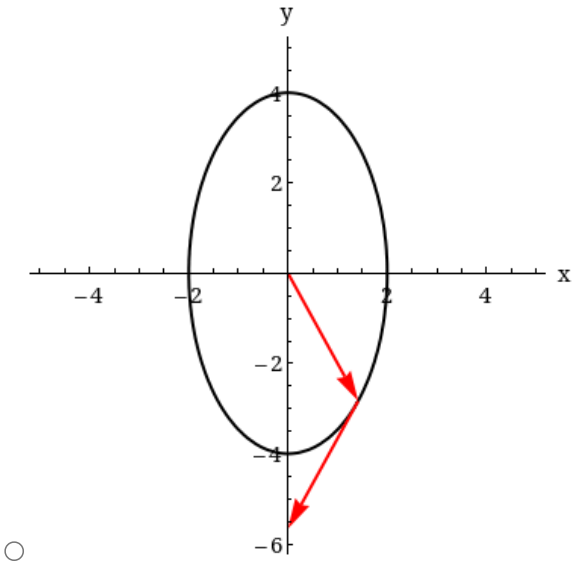
$\mathbf{r}(t) = 4\sin(t)\mathbf{i} - 2\cos(t)\mathbf{j}$

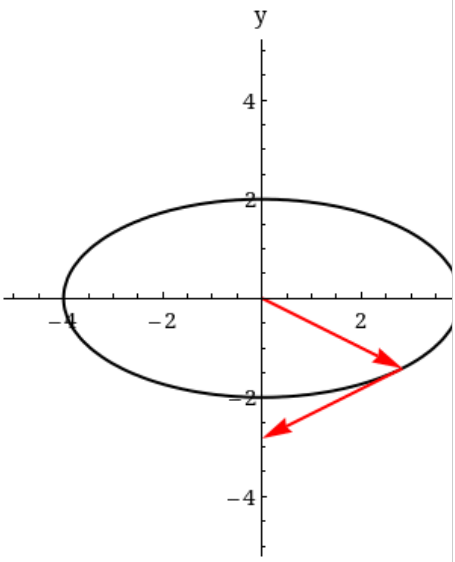
(a) Find $\mathbf{r}'(t)$.

(b) Sketch the plane curve together with position vector $\mathbf{r}(t)$ and the tangent vector $\mathbf{r}'(t)$ for the given value of $t = 3\pi/4$.









11. Question Details SCalcET9M 13.2.010. [4784350]

Find the derivative, $\mathbf{r}'(t)$, of the vector function.

$\mathbf{r}(t) = \langle e^{-t}, 8t - t^3, \ln(t) \rangle$

$\mathbf{r}'(t) =$

12. Question Details SCalcET9M 13.2.014. [5093884]

Find the derivative, $\mathbf{r}'(t)$, of the vector function.

$$\mathbf{r}(t) = \cos^2(at)\,\mathbf{i} + te^{bt}\mathbf{j} + \sin^2(ct)\,\mathbf{k}$$

$\mathbf{r}'(t) =$

13. Question Details SCalcET9M 13.2.017. [4784247]

Find the unit tangent vector $\mathbf{T}(t)$ at the point with the given value of the parameter t.

$$\mathbf{r}(t) = \left\langle t^2 - 4t,\ 1 + 5t,\ \frac{1}{3}t^3 + \frac{1}{2}t^2 \right\rangle, \quad t = 4$$

$\mathbf{T}(4) =$

14. Question Details SCalcET9M 13.2.018. [5093401]

Find the unit tangent vector $\mathbf{T}(t)$ at the point with the given value of the parameter t.

$$\mathbf{r}(t) = \left\langle \tan^{-1}(t),\ 2e^{3t},\ 18te^{t} \right\rangle, \quad t = 0$$

$\mathbf{T}(0) =$

15. Question Details SCalcET9M 13.2.019.EP. [5093533]

Consider the following vector function.

$$\mathbf{r}(t) = \cos(t)\,\mathbf{i} + 6t\mathbf{j} + 4\sin(2t)\,\mathbf{k}$$

Find each of the following.

$\mathbf{r}'(t) =$

$\mathbf{r}'(0) =$

$|\mathbf{r}'(0)| =$

Find the unit tangent vector $\mathbf{T}(t)$ at the point with the given value of the parameter t.

$$\mathbf{r}(t) = \cos(t)\,\mathbf{i} + 6t\mathbf{j} + 4\sin(2t)\,\mathbf{k}, \quad t = 0$$

$\mathbf{T}(0) =$

16. Question Details SCalcET9M 13.2.021. [5093262]

Find the unit tangent vector $\mathbf{T}(t)$ at the given point on the curve.

$$\mathbf{r}(t) = \left\langle t^3 + 1,\ 3t - 7,\ \frac{5}{t} \right\rangle, \quad (2, -4, 5)$$

17. Question Details SCalcET9M 13.2.022. [5093805]

Find the unit tangent vector $\mathbf{T}(t)$ at the given point on the curve.

$$\mathbf{r}(t) = \sin(t)\,\mathbf{i} + 8t\,\mathbf{j} + \cos(t)\,\mathbf{k}, \quad (0, 0, 1)$$

18. Question Details SCalcET9M 13.2.023. [5093333]

If $\mathbf{r}(t) = \langle t^8, t, t^2 \rangle$, find $\mathbf{r}'(t)$, $\mathbf{T}(1)$, $\mathbf{r}''(t)$, and $\mathbf{r}'(t) \times \mathbf{r}''(t)$.

$\mathbf{r}'(t)$ =

$\mathbf{T}(1)$ =

$\mathbf{r}''(t)$ =

$\mathbf{r}'(t) \times \mathbf{r}''(t)$ =

19. Question Details SCalcET9M 13.2.024. [5093836]

If $\mathbf{r}(t) = \langle e^{5t}, e^{-6t}, t \rangle$, find $\mathbf{r}'(0)$, $\mathbf{T}(0)$, $\mathbf{r}''(0)$, and $\mathbf{r}'(0) \times \mathbf{r}''(0)$.

$\mathbf{r}'(0)$ =

$\mathbf{T}(0)$ =

$\mathbf{r}''(0)$ =

$\mathbf{r}'(0) \times \mathbf{r}''(0)$ =

20. Question Details SCalcET9M 13.2.027.EP. [5093606]

Consider following parametric equations for a curve.

$$x = e^{-7t}\cos(7t), \quad y = e^{-7t}\sin(7t), \quad z = e^{-7t}$$

Determine the value of t that corresponds to the point (1, 0, 1).

$t =$

Write a vector equation $\mathbf{r}(t)$ for the given curve.

$\mathbf{r}(t) =$

Find $\mathbf{r}'(t)$.

$\mathbf{r}'(t) =$

Find parametric equations for the tangent line to the curve with the given parametric equations at the specified point.

$$x = e^{-7t}\cos(7t), \quad y = e^{-7t}\sin(7t), \quad z = e^{-7t}, \quad (1, 0, 1)$$

$(x(t), y(t), z(t)) = \left(\quad\right)$

21. Question Details SCalcET9M 13.2.030. [5093281]

Find the point on the curve $\mathbf{r}(t) = \langle 8\cos(t), 8\sin(t), e^t \rangle$, $0 \le t \le \pi$, where the tangent line is parallel to the plane $\sqrt{3}x + y = 1$.

$(x, y, z) = \left(\quad\right)$

22. Question Details SCalcET9M 13.3.019. [4783820]

Consider the following vector function.

$$\mathbf{r}(t) = \langle 6t^2, \sin(t) - t\cos(t), \cos(t) + t\sin(t) \rangle, \quad t > 0$$

(a) Find the unit tangent and unit normal vectors $\mathbf{T}(t)$ and $\mathbf{N}(t)$.

$\mathbf{T}(t) =$

$\mathbf{N}(t) =$

(b) Use this formula to find the curvature.

$\kappa(t) =$

23. Question Details SCalcET9M 13.4.003. [4784260]

Find the velocity, acceleration, and speed of a particle with the given position function.

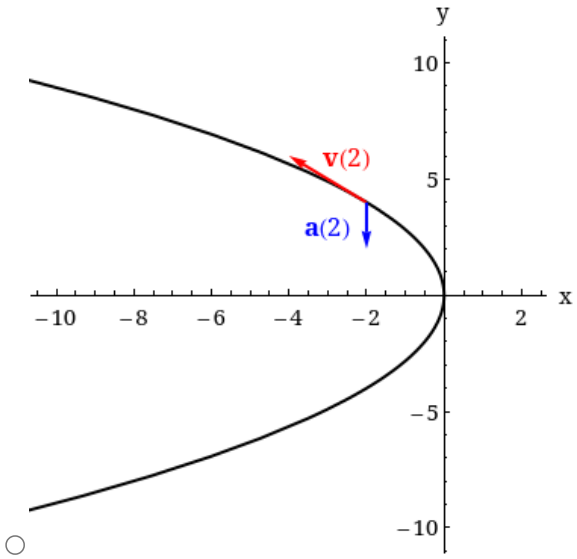
$$\mathbf{r}(t) = \left\langle -\frac{1}{2}t^2, 2t \right\rangle$$

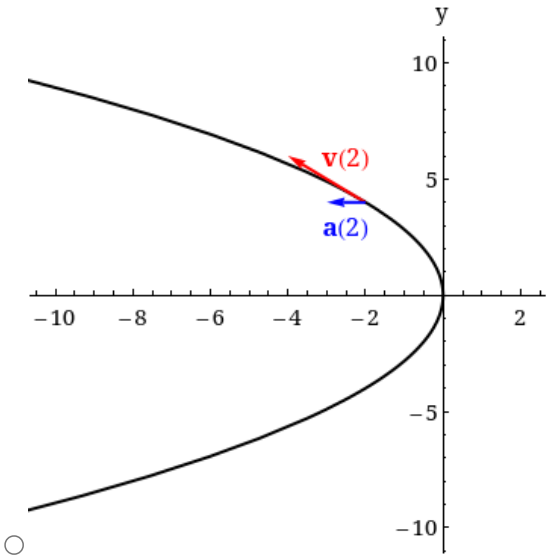
$\mathbf{v}(t)$ =

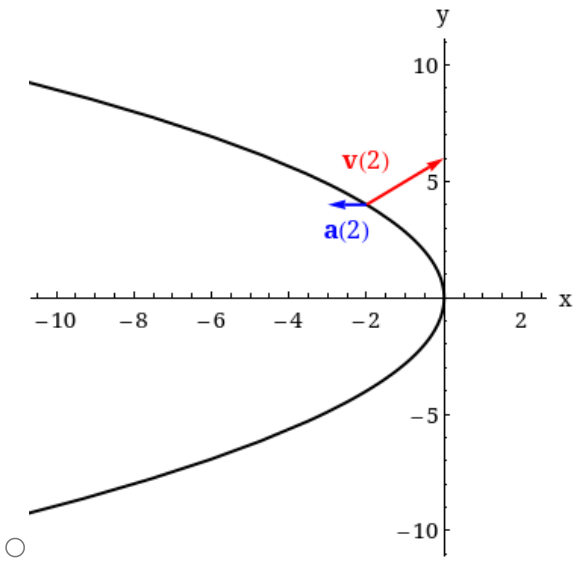
$\mathbf{a}(t)$ =

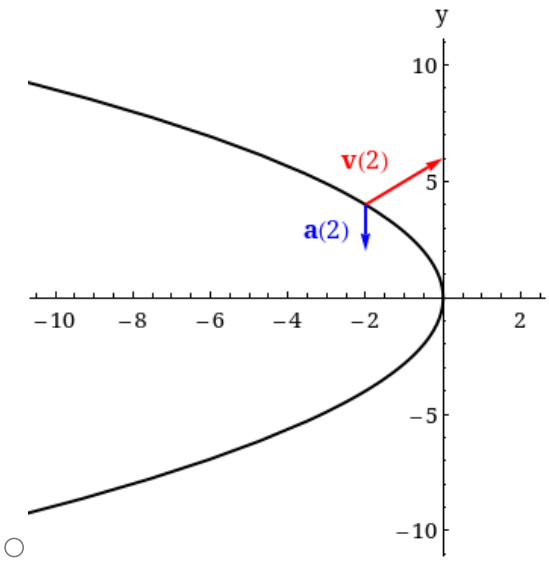
$|\mathbf{v}(t)|$ =

Sketch the path of the particle and draw the velocity and acceleration vectors for $t = 2$.









24. Question Details SCalcET9M 13.4.004. [5093288]

Find the velocity, acceleration, and speed of a particle with the given position function.

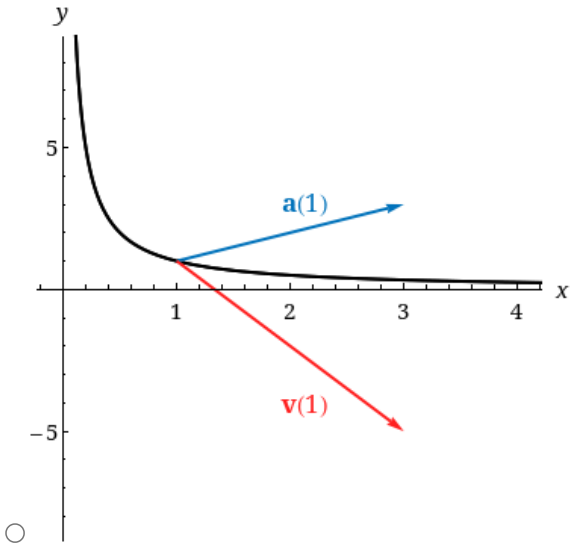
$$\mathbf{r}(t) = \left\langle t^2, \frac{1}{t^2} \right\rangle$$

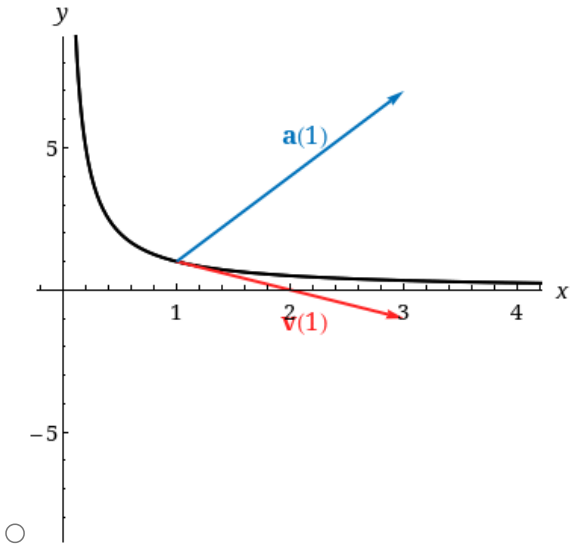
$\mathbf{v}(t)$ =

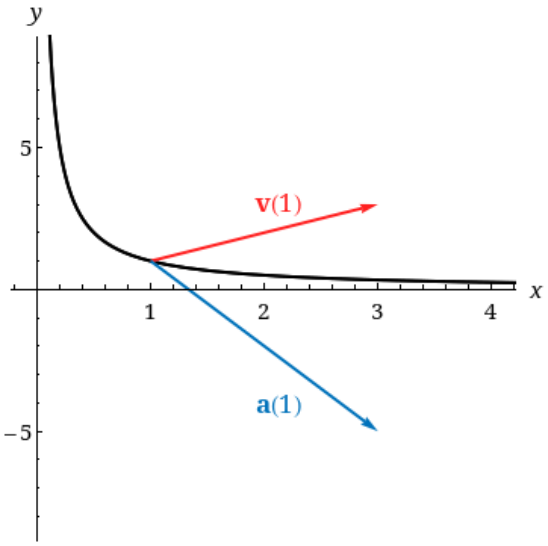
$\mathbf{a}(t)$ =

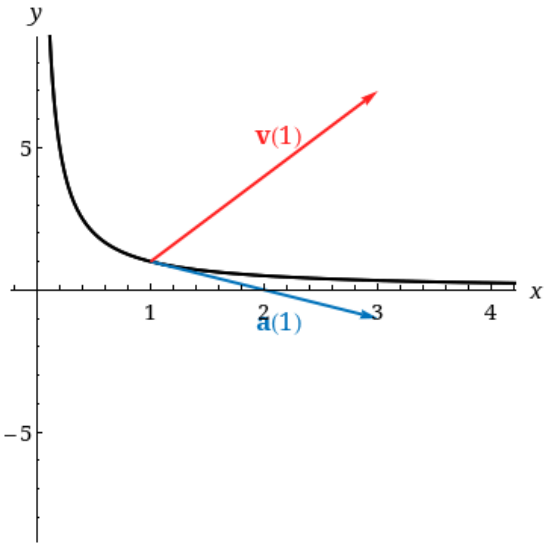
$|\mathbf{v}(t)|$ =

Sketch the path of the particle and draw the velocity and acceleration vectors for $t = 1$.






y
5
v(1)
x
1
2
3
4
a(1)
−5


y
5
v(1)
x
1
2
3
4
a(1)
−5

25. Question Details SCalcET9M 13.4.005. [4784391]

Find the velocity, acceleration, and speed of a particle with the given position function.

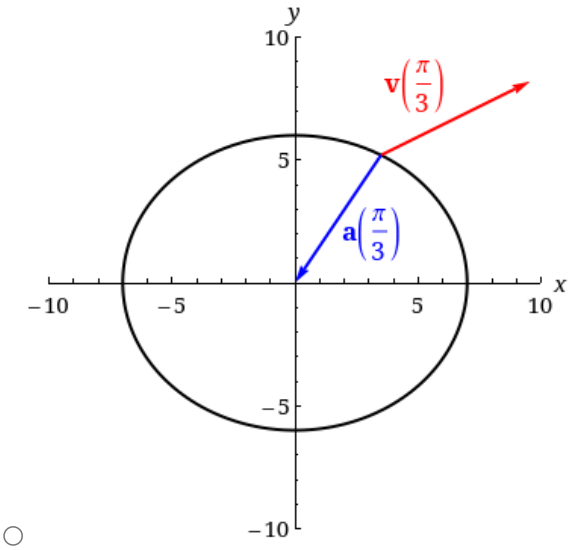
$\mathbf{r}(t) = 7\cos(t)\mathbf{i} + 6\sin(t)\mathbf{j}$

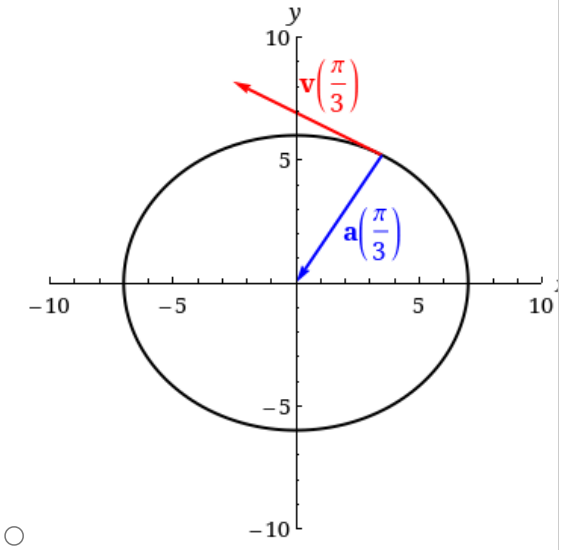
$\mathbf{v}(t) =$

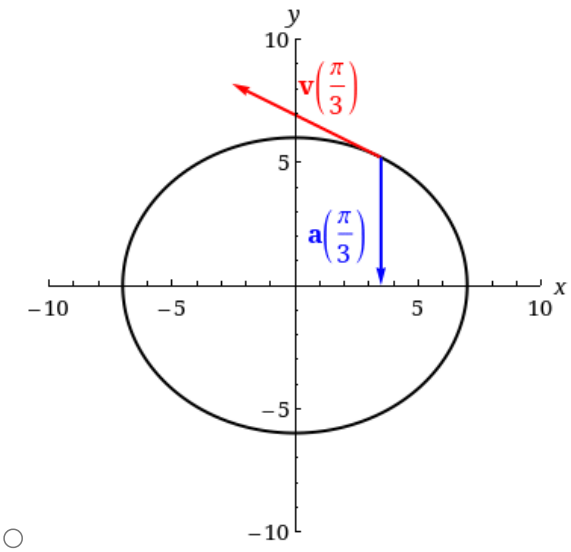
$\mathbf{a}(t) =$

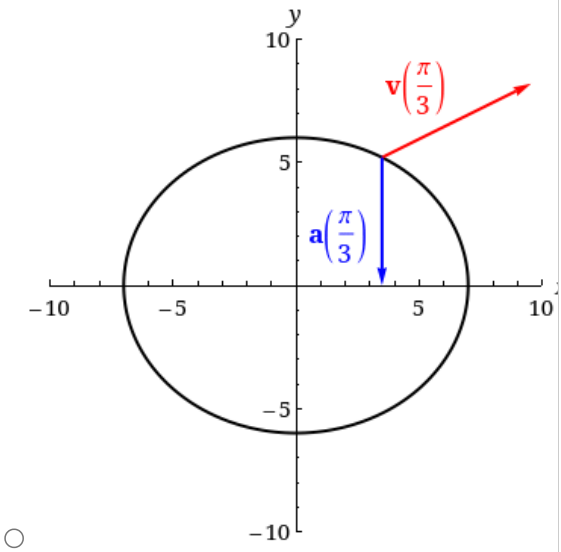
$|\mathbf{v}(t)| =$

Sketch the path of the particle and draw the velocity and acceleration vectors for $t = \frac{\pi}{3}$.









26. Question Details SCalcET9M 13.4.018. [4783857]

(a) Find the position vector of a particle that has the given acceleration and the specified initial velocity and position.

$\mathbf{a}(t) = 11t\mathbf{i} + e^{t}\mathbf{j} + e^{-t}\mathbf{k}, \quad \mathbf{v}(0) = \mathbf{k}, \quad \mathbf{r}(0) = \mathbf{j} + \mathbf{k}$

$\mathbf{r}(t) =$

(b) On your own using a computer, graph the path of the particle.

27. Question Details SCalcET9M 13.4.019.MI. [4783921]

The position function of a particle is given by $\mathbf{r}(t) = \langle t^2, 5t, t^2 - 16t \rangle$. When is the speed a minimum?

$t =$ ☐

28. Question Details SCalcET9M 13.4.022. [5093897]

Show that if a particle moves with constant speed, then the velocity and acceleration vectors are orthogonal.

Let the velocity vector be $\mathbf{v}(t)$ and the acceleration vector be $\mathbf{a}(t)$, and let $\mathbf{v}(t) = c$ (a constant). Since $\mathbf{v}(t) \cdot \mathbf{v}(t) = |\mathbf{v}(t)|^2 =$ ---Select--- which is a constant, the formula

$$\frac{d}{dt}\left[\mathbf{u}(t) \cdot \mathbf{v}(t)\right] = \mathbf{u}'(t) \cdot \mathbf{v}(t) + \mathbf{u}(t) \cdot \mathbf{v}'(t)$$

gives the following.

- ○ $2c = \frac{d}{dt}\left[\mathbf{v}(t) \cdot \mathbf{v}(t)\right] = \mathbf{a}(t) \cdot \mathbf{v}(t) + \mathbf{v}(t) \cdot \mathbf{a}(t) = (\mathbf{a}(t) \cdot \mathbf{v}(t))^2$
- ○ $0 = \frac{d}{dt}\left[\mathbf{v}(t) \cdot \mathbf{v}(t)\right] = \mathbf{a}(t) \cdot \mathbf{v}(t) + \mathbf{v}(t) \cdot \mathbf{a}(t) = 2\mathbf{a}(t) \cdot \mathbf{v}(t)$
- ○ $1 = \frac{d}{dt}\left[\mathbf{v}(t) \cdot \mathbf{v}(t)\right] = \mathbf{a}(t) \cdot \mathbf{v}(t) + \mathbf{v}(t) \cdot \mathbf{a}(t) = (\mathbf{a}(t) \cdot \mathbf{v}(t))^2$
- ○ $0 = \frac{d}{dt}\left[\mathbf{v}(t) \cdot \mathbf{v}(t)\right] = \mathbf{a}(t) \cdot \mathbf{v}(t) + \mathbf{v}(t) \cdot \mathbf{a}(t) = (\mathbf{a}(t) \cdot \mathbf{v}(t))^2$
- ○ $1 = \frac{d}{dt}\left[\mathbf{v}(t) \cdot \mathbf{v}(t)\right] = \mathbf{a}(t) \cdot \mathbf{v}(t) + \mathbf{v}(t) \cdot \mathbf{a}(t) = 2\mathbf{a}(t) \cdot \mathbf{v}(t)$
- ○ $2c = \frac{d}{dt}\left[\mathbf{v}(t) \cdot \mathbf{v}(t)\right] = \mathbf{a}(t) \cdot \mathbf{v}(t) + \mathbf{v}(t) \cdot \mathbf{a}(t) = 2\mathbf{a}(t) \cdot \mathbf{v}(t)$

Thus, $\mathbf{a}(t) \cdot \mathbf{v}(t) =$ ☐, which says that $\mathbf{a}(t)$ is orthogonal to $\mathbf{v}(t)$.

29. Question Details SCalcET9M 13.3.013.MI. [4783846]

Let C be the curve of intersection of the parabolic cylinder $x^2 = 2y$, and the surface $3z = xy$. Find the exact length of C from the origin to the point $\left(5, \frac{25}{2}, \frac{125}{6}\right)$.

30. Question Details SCalcET9M 13.3.013.MI.SA. [4784394]

This question has several parts that must be completed sequentially. If you skip a part of the question, you will not receive any points for the skipped part, and you will not be able to come back to the skipped part.

Tutorial Exercise

Let C be the curve of intersection of the parabolic cylinder $x^2 = 2y$, and the surface $3z = xy$. Find the exact length of C from the origin to the point $\left(1, \frac{1}{2}, \frac{1}{6}\right)$.

31. Question Details SCalcET9M 13.3.014.EP. [5093530]

Consider the following.

the length of the curve of intersection of the cylinder $9x^2 + y^2 = 9$ and the plane $x + y + z = 6$

Let C be the curve of intersection. Using t as the parameter where $0 \le t \le 2\pi$, write parametric equations for C and give the corresponding vector equation $\mathbf{r}(t)$.

$\mathbf{r}(t) =$

Find $\mathbf{r}'(t)$ and $|\mathbf{r}'(t)|$.

$\mathbf{r}'(t) =$

$|\mathbf{r}'(t)| =$

Find the length of C. (Round your answer to four decimal places.)

32. Question Details SCalcET9M 13.4.039. [4783875]

Find the tangential and normal components of the acceleration vector.

$\mathbf{r}(t) = \cos(t)\,\mathbf{i} + \sin(t)\,\mathbf{j} + t\,\mathbf{k}$

$a_T =$

$a_N =$

Assignment Details